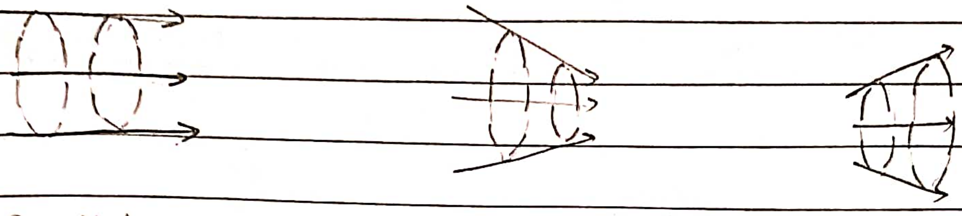
GEOMETRICAL OPTICS

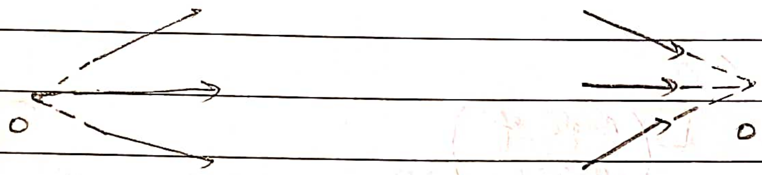
05/09/2023

- Ray — Directed line
- Beam — Bunch of rays.



Parallel Converging Diverging

- Pt object — Pt. of intersection of incident rays.

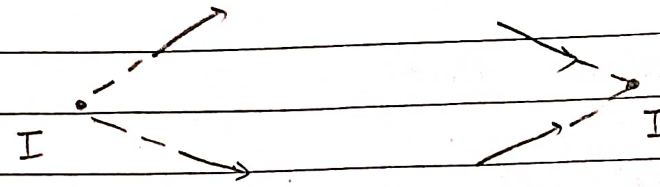


Real object (Diverging rays) Virtual object (Converging rays)

Eyes can only collect diverging rays.
So we can only see real objects.

* Principle of Geo. Optics — Ray of light travels in a straight line in one homogenous medium.

- Image — Pt. of intersection of reflected or refracted rays.



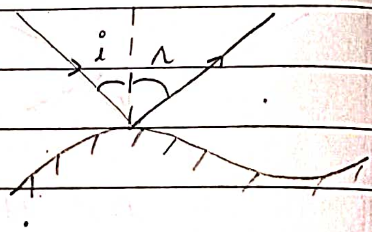
Virtual image   Real image

LAWS OF REFLECTION

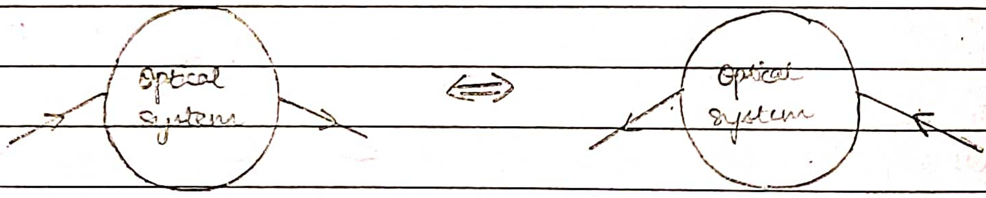
1. Incident ray, reflected ray & normal to mirror at pt. of incidence are coplanar (in plane of incidence)

2. $\angle i = \angle r$

(Angle of incidence)   (Angle of reflection)



- Law of reciprocity of light

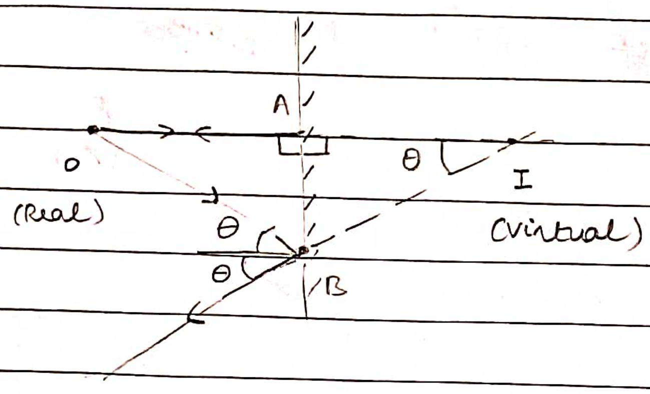


Path   Reciprocal path

OR

If object is placed at its image's initial post., new image is formed at the object's initial post.

PLANE MIRROR



Since rays from virtual image are diverging, we are able to see our image in the mirror!

$$OA = IA$$
$$OB = IB$$
$$\angle OBA = \angle IBA$$

⇒ Mirror ⊥ bisects the line joining Object & Image

&

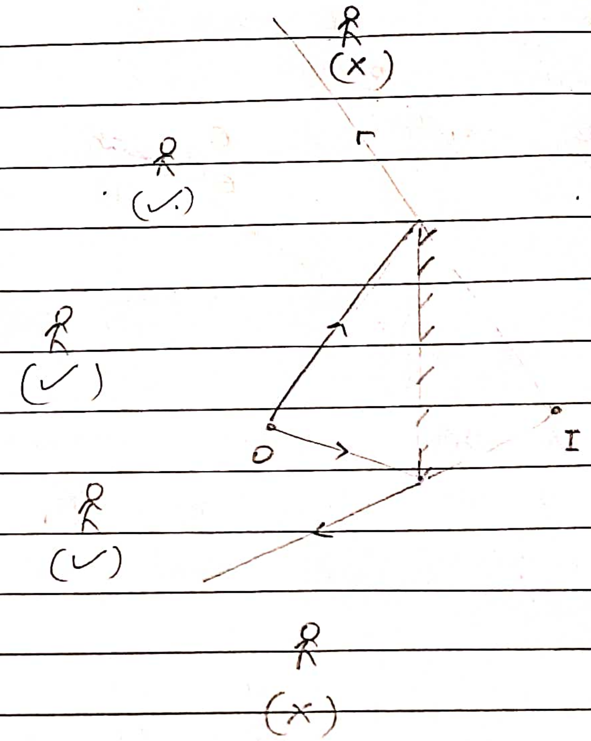
Mirror lies on the ∠ bisector of incident rays & its extended reflected ray

Real object ⟶ Virtual image
virtual object ⟶ Real image

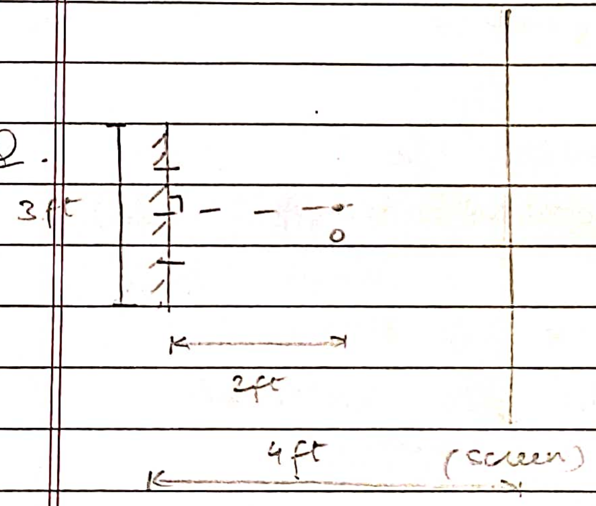
NOTE: 1. Lateral Inversion occurs in Plane mirror i.e
Obj's Right = Img's Left

2. Size (Obj) = Size (Img.)

- <u>Field view of image</u> — Region in which image of object is visible to observer

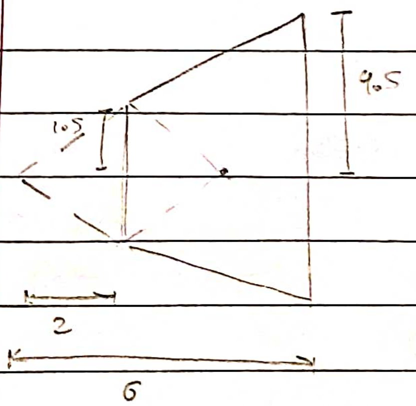


<u>Q.</u>



Find length of reflected patch on the screen

<u>A.</u>



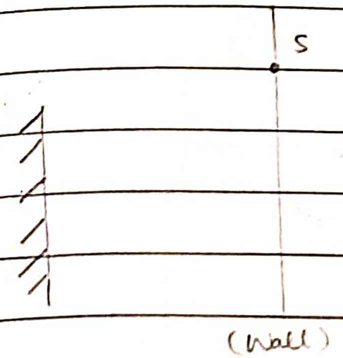
$\Rightarrow$ Total $= 2(4.5)$

$= \underline{9 \text{ ft}}$

NOTE: In the above Q, if object is not placed on ⊥ bisector, the reflected patch's length will remain same

It depends only on the dist. of object & the screen from plane mirror.

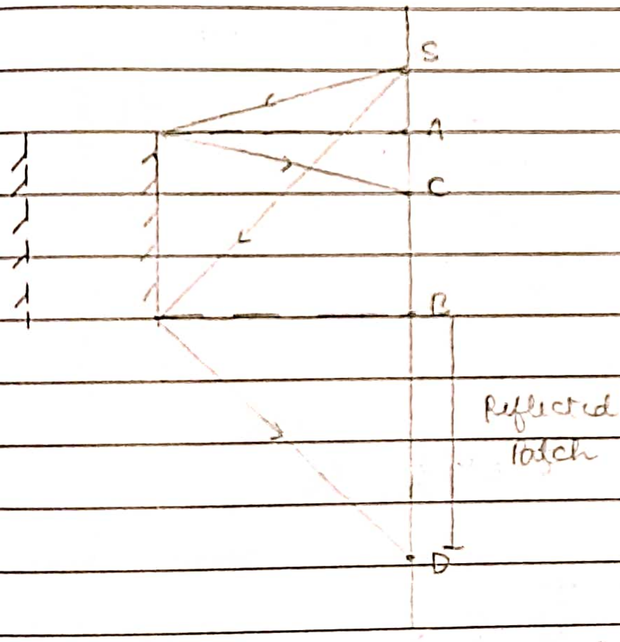
Q. If the mirror is moved closer to the wall, find the change in the
a) size of reflected patch
b) Posit. of reflected patch



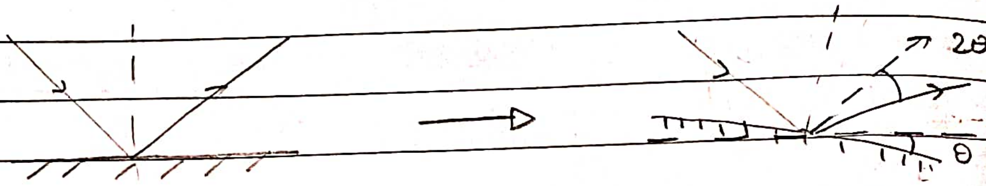
A. Even if mirror moves, A & B are fixed

Since $SA = AC$ & $SB = BD$

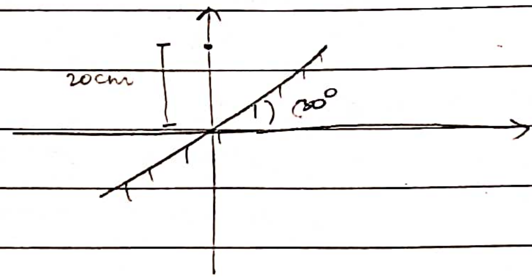
Therefore, size & posit of the reflected patch is const.



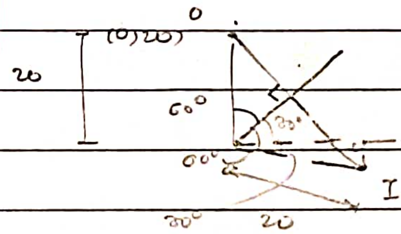
NOTE: This is only because the source is on the screen



If plane rotated about an axis in its plane by θ, the reflected ray is rotated in the same dirxn by 2θ

Q. 

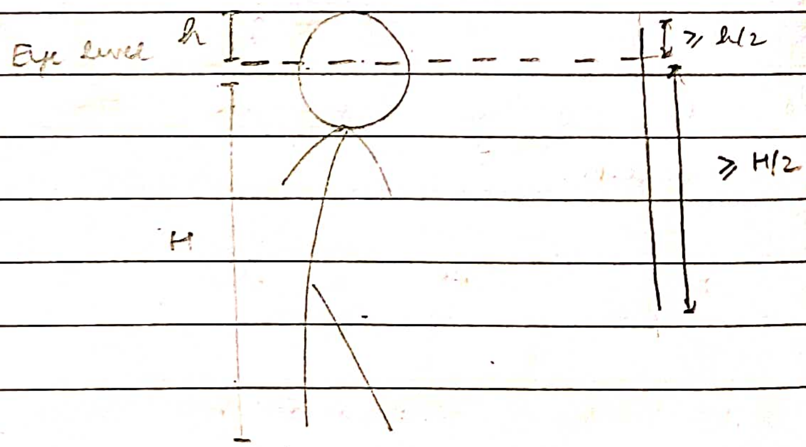
Find coordinates of image

A. 

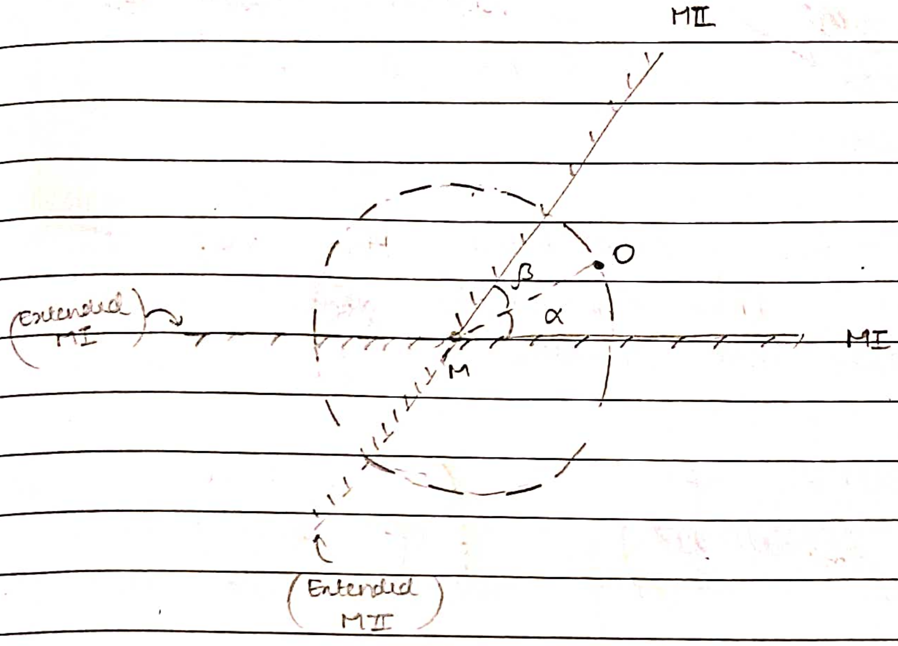
I (20cos(−30°), 20sin(−30°))

$I \equiv (10\sqrt{3}, -10)$

For seeing image of oneself in mirror

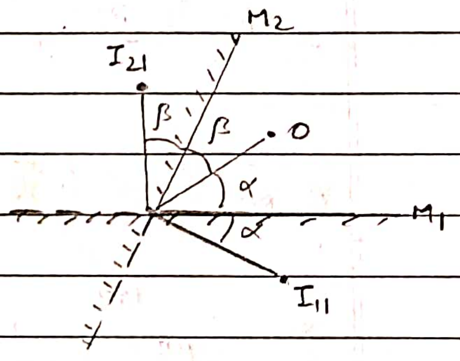


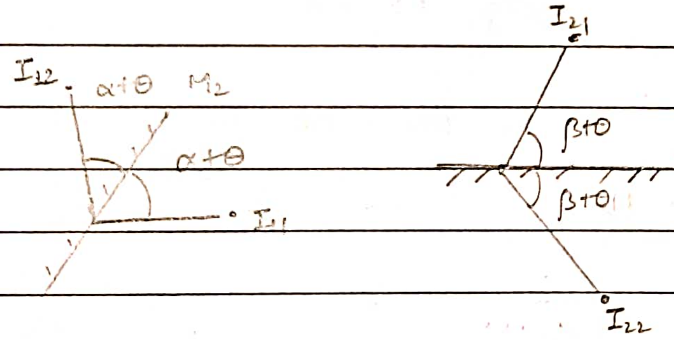
→ Multiple Mirrors



All images lie on the circle since M from equidistant O & all its images

The first images in M_1 & M_2 (ie I_{11} & I_{21}) act as objects for M_2 & M_1 respectively



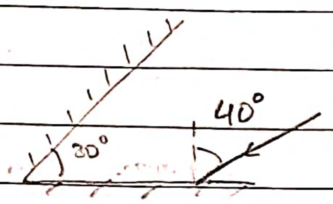


So,

	M_1		M_2	
I_{11}	α	$+\theta$	β	I_{21}
I_{12}	$\beta+\theta$	$+\theta$	$\alpha+\theta$	I_{22}
I_{13}	$\alpha+2\theta$		$\beta+2\theta$	I_{23}
⋮	⋮		⋮	

(as long as angles $< 180°$)

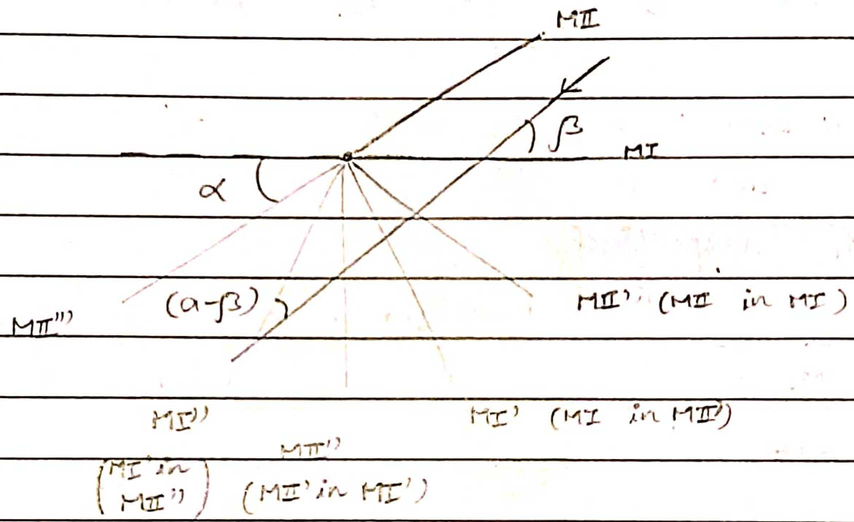
NOTE:
If $\alpha=\beta$,
last images formed by both the mirrors coincides

☆ Q. 

After how many reflections will the ray exit the mirrors

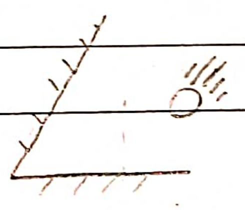
A. Either we can trace the ray, or we can take multiple reflections of one mirror into another

(Intersection of mirror & reflected ray) ≡ (Intersection of reflected mirror & incident ray)



Ray will reflect as long as

$$\alpha - \beta > 0 \Rightarrow \boxed{\alpha > \beta}$$

NOTE: 

This Q is eq. to the given mechanics problem

All collisions elastic, find #collisions

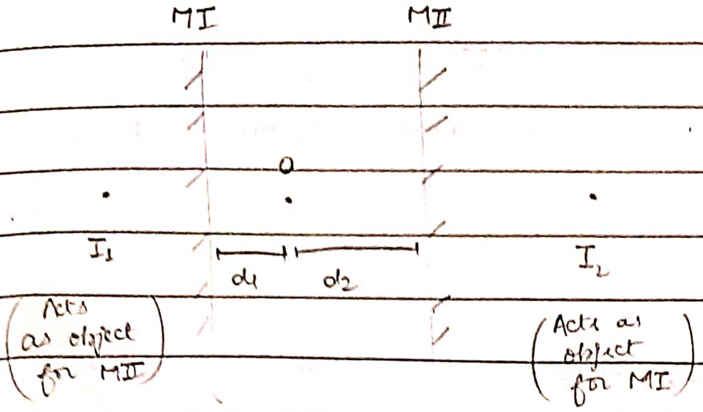
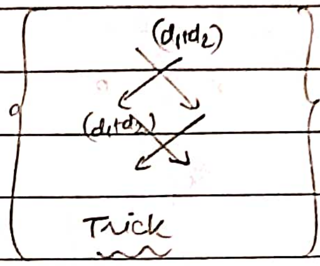


Image dist. from

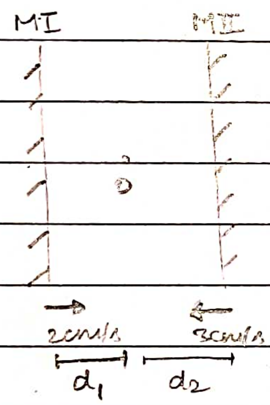
MI	MII
d_1	d_2
$d_2+(d_1+d_2)$	$d_1+(d_1+d_2)$
$d_1+2(d_1+d_2)$	$d_2+2(d_1+d_2)$



→ Vel. of object & image

$$\vec{v}_{IM(\perp)} = -\vec{v}_{OM(\perp)}$$

$$\vec{v}_{IM(\parallel)} = \vec{v}_{OM(\parallel)}$$



Q. Find vel. of 3rd image formed by MI

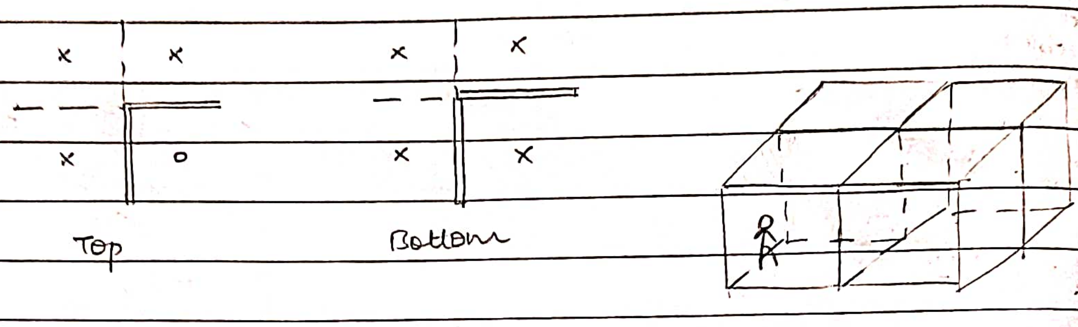
A. $s_{3\,(MI)} = -d_1 - 2(d_1+d_2) - d_1 \Rightarrow \vec{v}_3 = -4\dfrac{d(d_1)}{dt} - 2\dfrac{d(d_2)}{dt}$

$$= (-4)(-2) - 2(-3)$$

$$= \underline{14\text{ cm/s}}$$

Q. Two adj walls & ceiling are silvered. A person is standing ..... How many images are formed

A.



8 symm. placed images
Out of these, 1 is the original object
Hence 7 images

REFRACTION (Plane surface)

$$\mu = \left(\frac{\text{speed in vacuum}}{\text{speed in medium}}\right) = \frac{c}{v}$$

(Refractive index)

$\mu \geq 1$

When light travels from one medium to another, its freq. remains unchanged

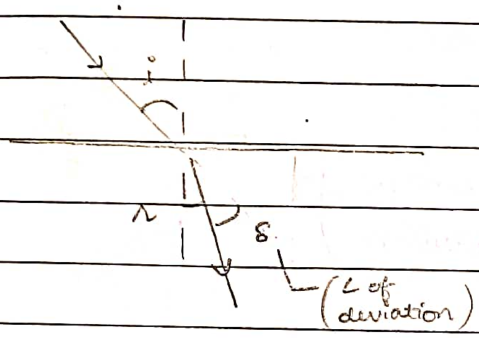
→ Laws of Refraction

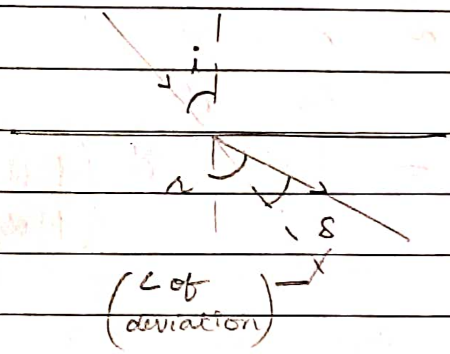
1. Incidence ray, refracted ray & normal to surface at pt. of incidence are <u>coplanar</u>

2. Snell's law

$$\boxed{\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{c_1}{c_2}}$$

	Rarer		Denser
μ:		$<$	
c_m:		$>$	





Rarer → Denser
(Towards normal)

Denser → Rarer
(Away from normal)

$\delta = 0 \Rightarrow \mu_1 = \mu_2$ <u>OR</u> $i = r = 0$

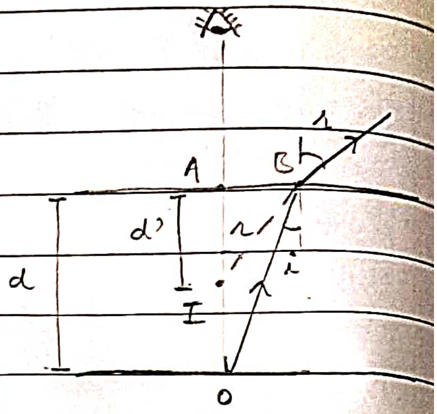
→ Apparent Depth (for normal vision)

⇓

Paraxial rays

$\Rightarrow (\sin i \sim \tan i = i)$

$$\tan i = \frac{AB}{OA} \quad \& \quad \tan r = \frac{AB}{IA}$$

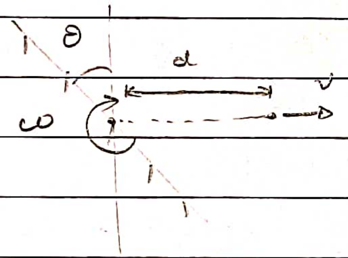


$$\Rightarrow \frac{\tan i}{\tan r} = \frac{IA}{OA} = \frac{d'}{d}$$

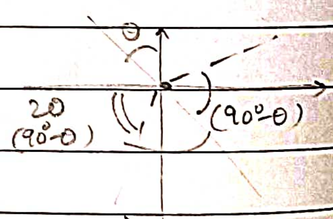
$$\sim \frac{\sin i}{\sin r} = \frac{d'}{d}$$

$$\Rightarrow \boxed{d' = d\left(\frac{\mu_{\text{observer medium}}}{\mu_{\text{object medium}}}\right)}$$

Q. Find v_{img}



A. Method I

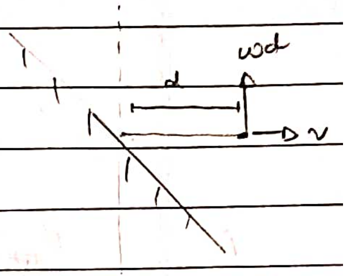


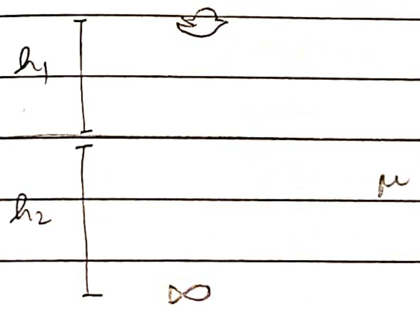
$$\vec{r}_{img} = \langle -d\cos 2\theta \quad -d\sin 2\theta \rangle$$

$$\vec{v}_{img} = \langle -v\cos 2\theta + 2d\sin 2\theta\,\omega \quad -v\sin 2\theta - 2d\cos 2\theta\,\omega \rangle$$

Method II

Let us place an observer at axis of rot^n rotating with 'ω'



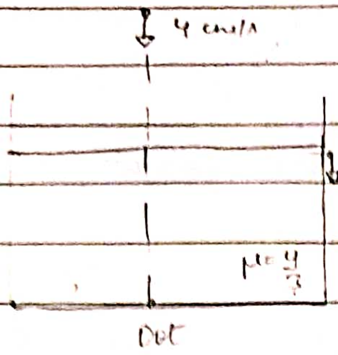


Q. Find dist at which
(i) fish will appear to bird
(ii) bird will appear to fish

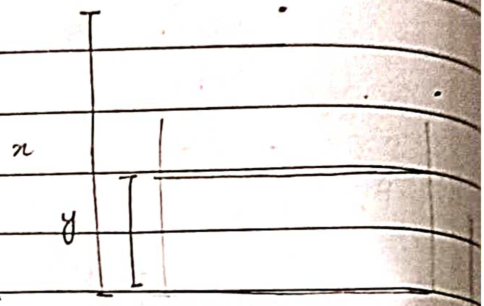
A. (i) $d = d_{air} + d_{water} = h_1 + h_2' = h_1 + \frac{h_2}{\mu}$

(ii) $d = d_{water} + d_{air} = h_2 + h_1' = h_2 + \mu h_1$

Q Find velocity of dot as observed by the observer



A $$d_{\substack{dot/\\observer}} = (x-y) + \frac{y}{\mu}$$



$$v_{\substack{dot/\\observer}} = \frac{dx}{dt} + \left(\frac{1}{\mu}-1\right)\frac{dy}{dt}$$

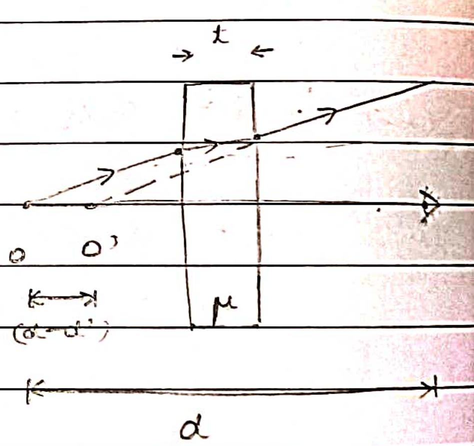
$$= -4 + \left(\frac{3}{4}-1\right)(-2)$$

$$= -3.5 \text{ cm/s}$$

- Refraction by Glass Slab

$$d' = \underbrace{(d-t)}_{\text{(Dist in air)}} + \frac{t}{\mu}$$

$$= d + \left(\frac{1}{\mu}-1\right)t$$



Apparent shift $= d - d'$

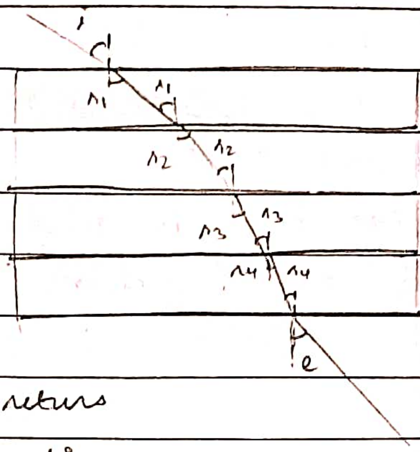
$$= t\left(1-\frac{1}{\mu}\right)$$

NOTE: If diverging rays → shift towards slab
converging rays → shift away from slab

$$\frac{\sin i}{\sin r_1} = \frac{\mu_1}{\mu}, \quad \frac{\sin r_1}{\sin r_2} = \frac{\mu_2}{\mu_1}$$

$$\frac{\sin r_2}{\sin r_3} = \frac{\mu_3}{\mu_2}, \quad \frac{\sin r_4}{\sin r_3} = \frac{\mu_4}{\mu_3}$$

$$\frac{\sin e}{\sin r_4} = \frac{\mu}{\mu_4}$$



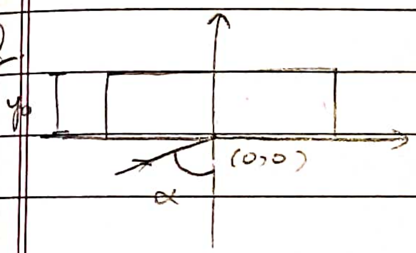
$\Rightarrow \quad \frac{\sin e}{\sin i} = 1 \quad \Rightarrow$ If light returns in same medium after refraction,

$i = e \Leftrightarrow$ Ray parallel

$$\frac{\sin i}{\sin r_2} = \frac{\mu_2}{\mu}, \quad \frac{\sin i}{\sin r_3} = \frac{\mu_3}{\mu}, \quad \frac{\sin i}{\sin r_4} = \frac{\mu_4}{\mu}.$$

$\Rightarrow$ $\angle$ of refraction on a surface is independent of any past refractions from any other surface.

Q.



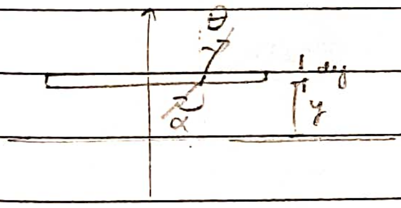
$\mu_y = \sqrt{1+y^{1/2}}$

a) Find eq^n of ray inside the slab

b) Find coordinates of the pt. at which light leaves the slab

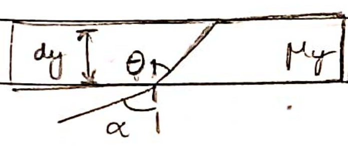
$\alpha \to 90^\circ$ (grazing incidence)

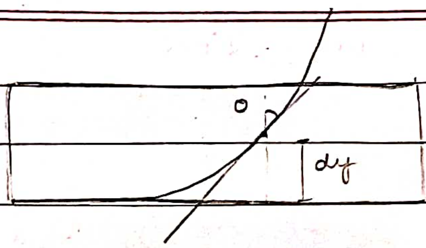
A.



$\sin \alpha = \mu_y \sin \theta$

$\Rightarrow \quad \sin \theta = \frac{1}{\sqrt{1+y^{1/2}}}$





(★ Ray is tangent to the curve)

$$\frac{dy}{dx} = \cot(\theta) = y^{1/4}$$

$$\Rightarrow \int \frac{dy}{y^{1/4}} = \int dx$$

$$\Rightarrow \frac{4}{3} y^{3/4} = x + C$$

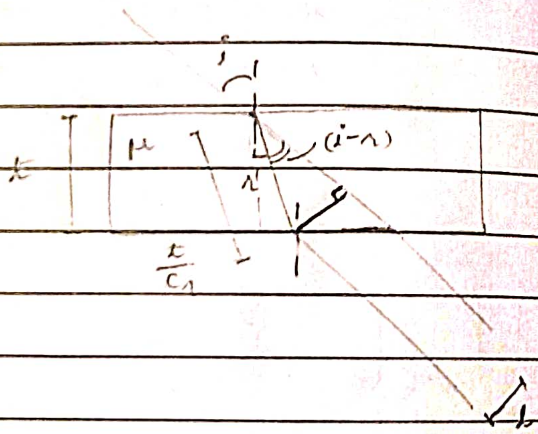
(0,0)

$$\Rightarrow 4y^{3/4} = 3x$$

Pt at which ray leaves the surface is $\left(\frac{4}{3} y_0^{3/4}, y_0\right)$

(Lateral Shift)

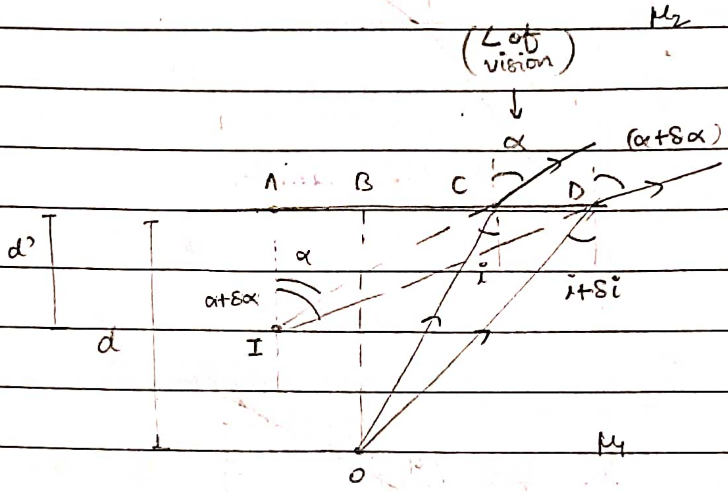
$$\boxed{l = \frac{t}{c_r} s(i-r)}$$



→ Apparent Depth (given an $\angle$ of vision)

$BC = d\,t_i$

$BD = d\,t_{(i+\delta i)}$



$AC = d'\,t_\alpha$

$AD = d'\,t_{(\alpha+\delta\alpha)}$

$CD = CD$

$\Rightarrow d\left(t_{(i+\delta i)} - t_i\right)$

$= d'\left(t_{(\alpha+\delta\alpha)} - t_\alpha\right)$

$$\Rightarrow \frac{d\, s_{\delta i}}{c_{(i+\delta i)}c_i} = \frac{d'\, s_{\delta\alpha}}{c_{(\alpha+\delta\alpha)}c_\alpha} \xrightarrow{\text{lim}} \frac{d\, di}{c_i^2} = \frac{d'\, d\alpha}{c_\alpha^2}$$

$$\Rightarrow d' = d\left(\frac{c_\alpha^2}{c_i^2}\right)\left(\frac{di}{d\alpha}\right)$$

By Snell's Law, $\mu_1 s_i = \mu_2 s_\alpha \Rightarrow \mu_1 c_i\, di = \mu_2 c_\alpha\, d\alpha$

$$\Rightarrow \frac{di}{d\alpha} = \frac{\mu_2 c_\alpha}{\mu_1 c_i}$$

$$\Rightarrow \boxed{d' = d\left(\frac{\mu_2}{\mu_1}\right)\left(\frac{c_\alpha^3}{c_i^3}\right)}$$

Date
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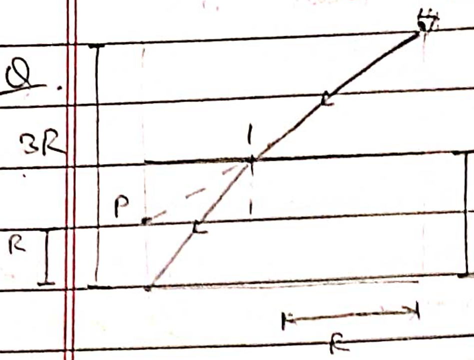
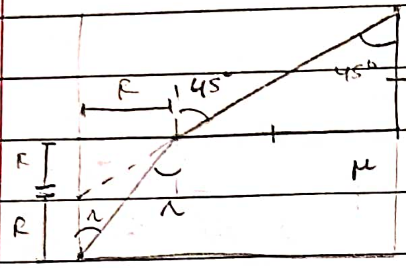
Q.



Bottom corner just visible on pouring liq. Find μ.

A.

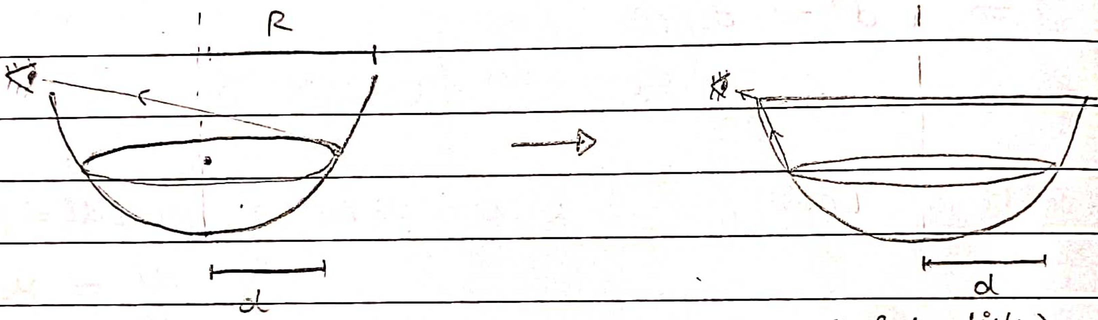


$$\sin i = \mu \sin r$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \mu \frac{1}{\sqrt{5}}$$

$$\Rightarrow \mu = \sqrt{\frac{5}{2}}$$

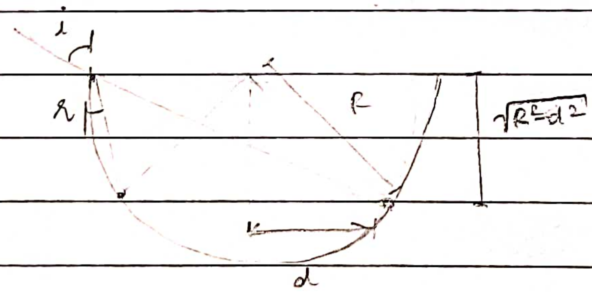
Q.



(Whole disk visible)

Find μ

A.



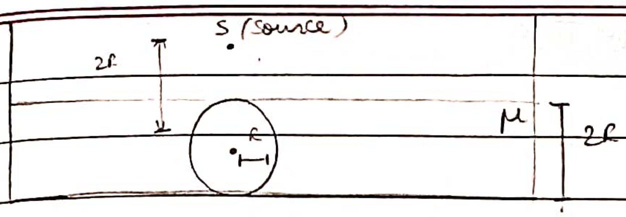
$$\tan i = \frac{R+d}{\sqrt{R^2-d^2}}$$

$$\tan r = \frac{R-d}{\sqrt{R^2-d^2}}$$

$$\sin i = \mu \sin r$$

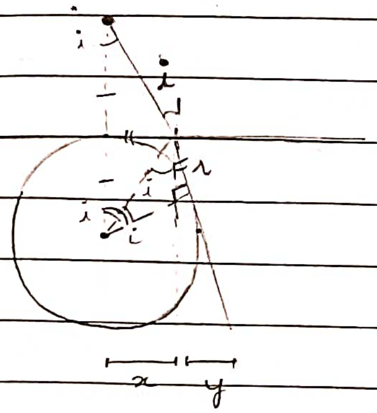
$$\Rightarrow \mu = \frac{\sin i}{\sin r} = \frac{R+d}{\sqrt{(R+d)^2+R^2-d^2}} \cdot \frac{\sqrt{(R-d)^2+R^2-d^2}}{R-d} = \sqrt{\frac{R+d}{R-d}}$$

Q



Find area of shadow

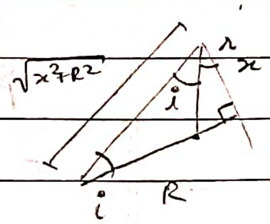
A



$\sin i = \mu \sin r$

$\Rightarrow \sin i = \mu \sin\left(\frac{\pi}{2} - 2i\right)$

$\Rightarrow \underline{\sin i = \mu \cos 2i}$



$i + (i+r) + \pi/2 = \pi$

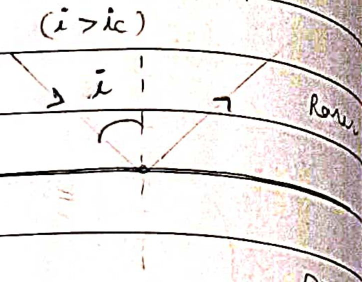
solve for i $\qquad \Rightarrow \underline{2i + r = \pi/2}$

$$\left.\begin{aligned} \tan(i+r) &= \left(\frac{R}{x}\right) \\ \tan r &= \left(\frac{y}{2R}\right) \\ \tan i &= \left(\frac{x}{R}\right) \end{aligned}\right\} \Rightarrow \quad l = x+y = R\tan i + 2R\tan r = \underline{R\tan i + \frac{2R}{\tan 2i}}$$

Use the value obtained for i to find 'l'.

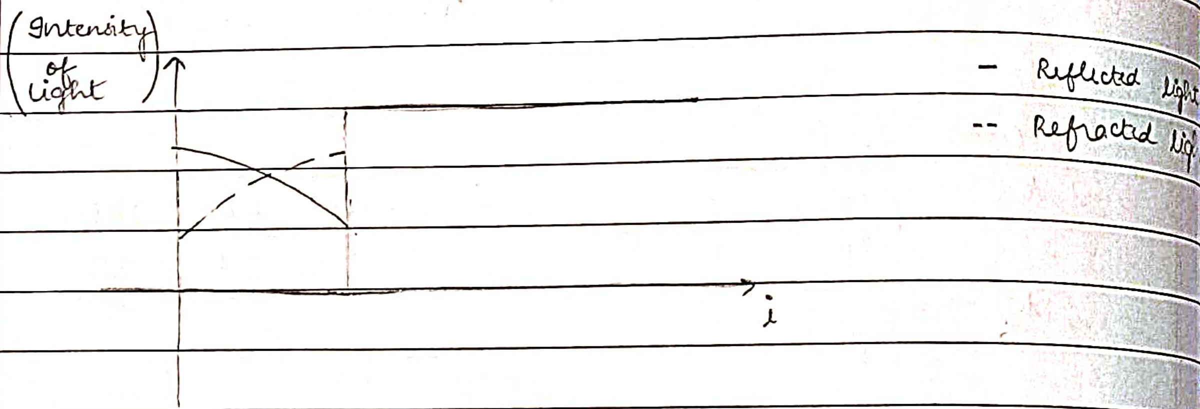
TOTAL INTERNAL REFLECTION

$i > i_c$
$\quad\llcorner$ (critical $\angle$)



$$\mu_1 \sin i_c = \mu_2 \sin 90^\circ$$

$$\Rightarrow \boxed{i_c = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)}$$



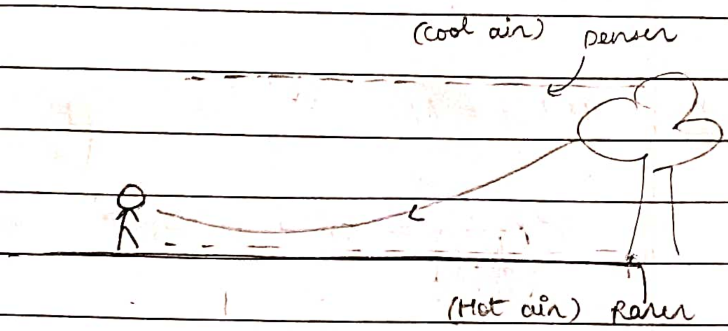
Fibre optical cable is based on TIR.
It is preferred over electrical wires for data communications because :-

1. Electrical wires suffer from EM interference
2. Large bandwidth (range of freq.)
3. Harder to steal signal

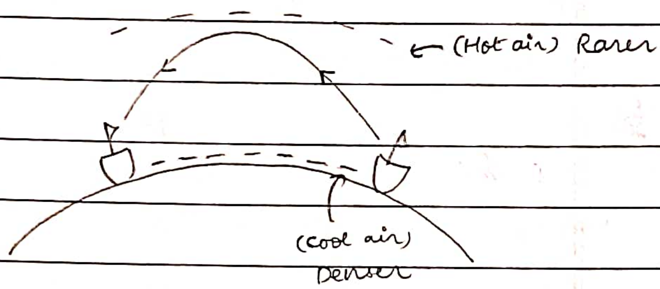
— Medical application — Endoscopy

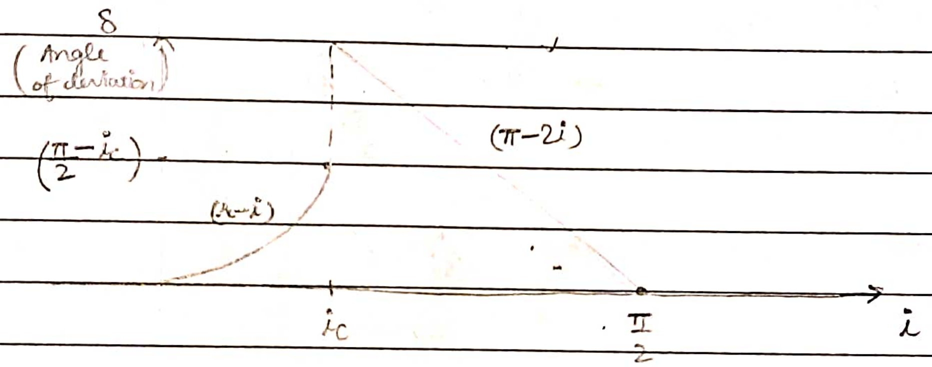
- Diamond shines because of TIR

- Mirage -



- Looming -





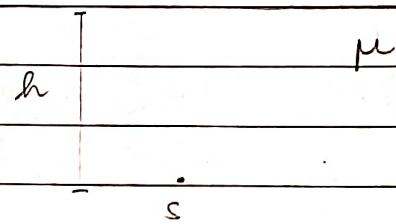
$$\delta = \begin{cases} (r - i), & i \le i_c \\ (\pi - 2i), & i > i_c \end{cases}$$

Q. δ observed to be same for $i = 30°$ & $75°$

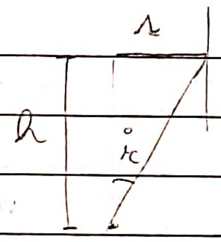
Find μ.

A. $\mu \sin 30° = \sin r \Rightarrow r = \sin^{-1}\left(\frac{\mu}{2}\right)$

$$\sin^{-1}\left(\frac{\mu}{2}\right) - 30° = 180° - 180° \Rightarrow \frac{\mu}{2} = \frac{\sqrt{3}}{2} \Rightarrow \mu = \sqrt{3}$$

Q. 

Find min. radius of disc s.t light does not escape.

A. 

$\tan i_c = \frac{r}{h}$ & $\mu \sin i_c = 1$

$\sin i_c = \frac{1}{\mu}$

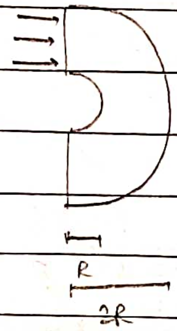
$$\Rightarrow r = h \frac{\frac{1}{\mu}}{\sqrt{1 - \frac{1}{\mu^2}}}$$

$$= \frac{h}{\sqrt{\mu^2 - 1}}$$

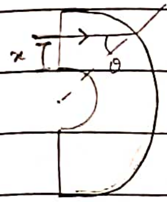
Q. In the above Q, if the disc were removed, what fraction of light produced by source escapes.

A. $$f = \frac{S.A\ (\text{cone})}{4\pi} = \frac{2\pi(1 - \cos i_c)}{4\pi} = \frac{1}{2}\left(1 - \frac{\sqrt{\mu^2 - 1}}{\mu}\right)$$

Q. Find min μ s.t light does not escape the cylinder from curved surface.



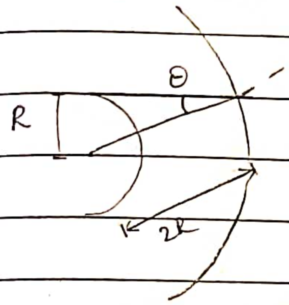
A.



For any x, $\theta > i_c$

θ_{min} at $x = 0$

If we ensure TIR for $x=0$, TIR is ensured $\forall x$



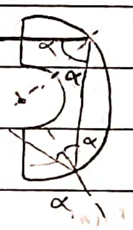
$\mu \sin i_c = 1 \Rightarrow i_c = \sin^{-1}\left(\frac{1}{\mu}\right)$

$\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$; $\theta \geq i_c$

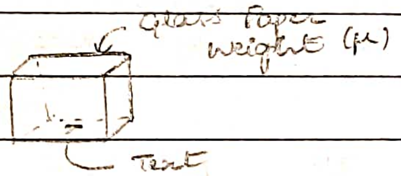
$\Rightarrow \underline{\mu \geq 2}$

NOTE Light will only escape through plane surface as,

$\alpha > \theta_c \Rightarrow$ TIR everywhere

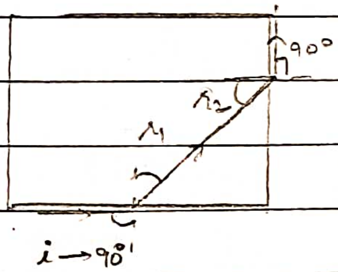


Q.



Find μ_{min} s.t text is not visible from vertical faces.

A.



$i \to 90°$, $r_1 \to \theta_c$, so $i\uparrow \Rightarrow r_1\uparrow \Rightarrow r_2\downarrow$
(by reversibility of light)

if for least r_2 TIR occurs

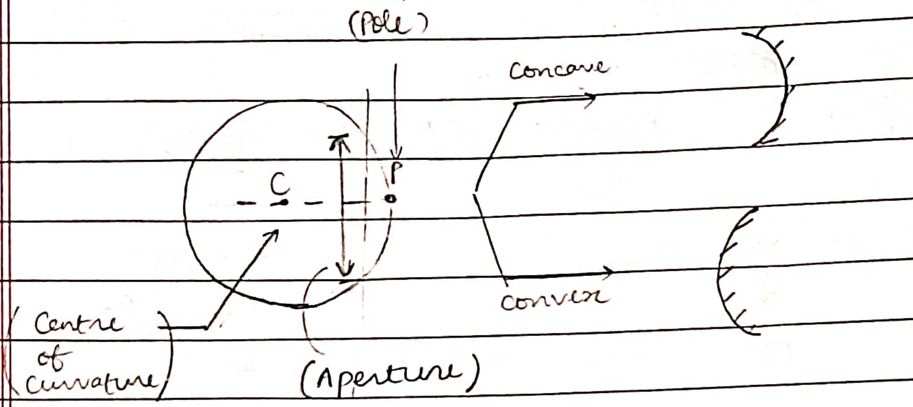
$\Rightarrow$ TIR occurs for all r_2

$r_{2(min)} = \frac{\pi}{2} - \theta_c \Rightarrow r_{2(min)} \geq \theta_c \Rightarrow \frac{\pi}{2} - \theta_c \geq \theta_c \Rightarrow \underline{\theta_c \leq \pi/4}$

$[\because r_1 \to \theta_c]$ $\Rightarrow \sin\theta_c \leq 1/\sqrt{2} \Rightarrow \underline{\mu \geq 1.41}$

12/09/2023

SPHERICAL SURFACES

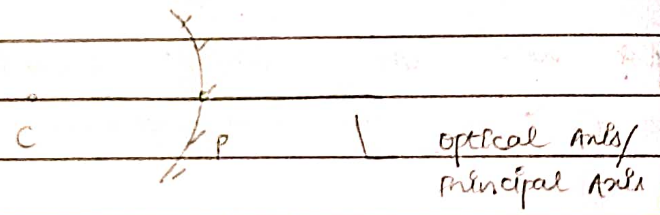


$ar(\text{Reflecting surface}) \propto (\text{Aperture})^2$

Apparture only affects brightness of image

$\text{Brightness (Intensity)} \propto (\text{Aperture})^2$

- Sign Convention

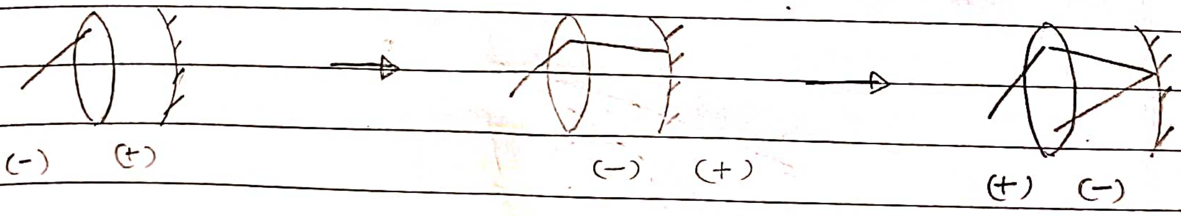


- Dist measured from P & along principal axis
- Dist in same dirn as incident ray > 0
  opp.. dirn as incident ray < 0

$\Downarrow$

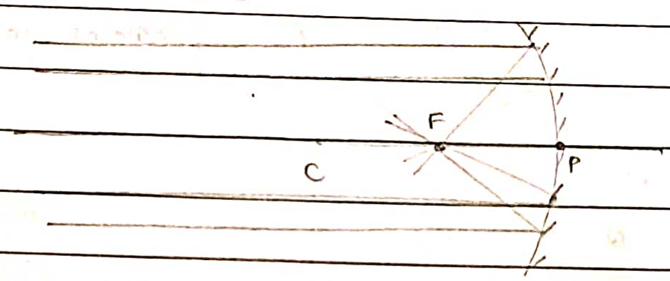
Real object's side < 0
Virtual object's side > 0

eg:-



We only study paraxial optics. i.e $\angle i$ is very small

Marginal rays - $\angle i$ very big

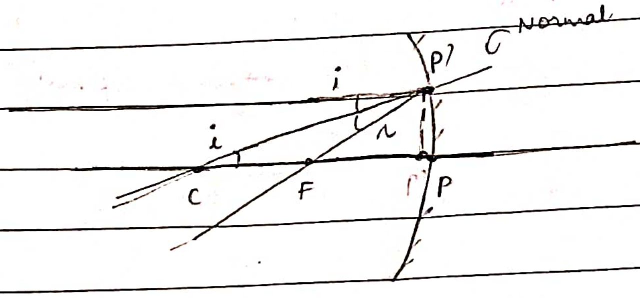


Paraxial rays produce bright image at a single pt. (focus)
Marginal rays produce several dull images at diff. pts. closer & closer to the pole.

This defect is called spherical abberation

To reduce this, we can use :-
- Mirrors of small aperture
- Parabolic mirror

• Focal length - FP



Derivation

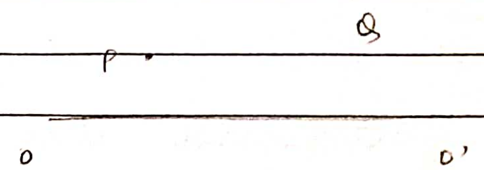
$i = r \Rightarrow CF = FP' \sim FP$

$CP = CF + FP \sim FP + FP \Rightarrow FP = CP/2$

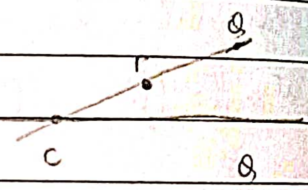
$\Rightarrow \underline{f = \frac{R}{2}}$

(for paraxial rays)

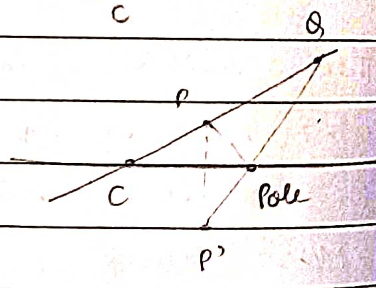
Q. Q is image of P using ray diagram, locate P, F & C. Also determine the nature of mirror.



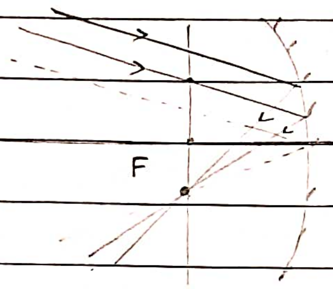
A. PQ meets OO' at C



For locating pole, take reflection of P about OO'. Line joining P'Q is pole



Rays which are || but not to the principal axis meet on the focal plane.

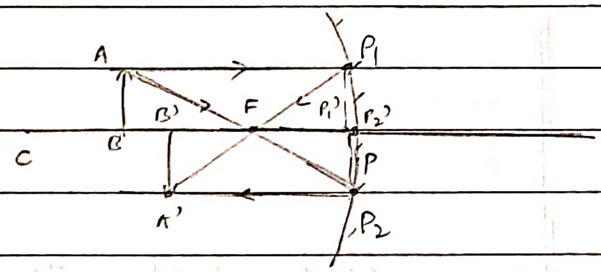


→ Mirror formula

$$\boxed{\frac{1}{f} = \frac{1}{v} + \frac{1}{u}}$$

$$\frac{P_2P_2'}{AB} = \frac{PF}{FB}$$

$$\frac{P_1P_1'}{A'B'} = \frac{PF}{FB'}$$



$P_2P_2' = A'B'$ & $P_1P_1' = AB$ (rectangle) $\Rightarrow \frac{PF}{FB} = \frac{FB'}{PF} \Rightarrow PF^2 = FB \cdot FB'$

$\Rightarrow PF^2 = (BP - FP)(B'P - FP) \Rightarrow \boxed{f^2 = (v-f)(u-f)}$

$\Rightarrow BP \cdot B'P = BP \cdot FP + FP \cdot B'P$

$\Rightarrow \frac{1}{FP} = \frac{1}{B'P} + \frac{1}{BP} \Rightarrow \frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

$m = \left(\frac{\text{length of image}}{\text{length of object}}\right)$

For object ⊥ to principal axis

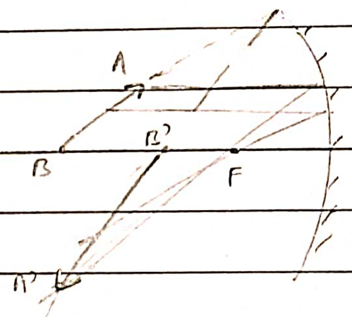
(image is inverted)

$$m = \frac{-A'B'}{AB} = -\left(\frac{PF + B'F}{BF+FP}\right) = -\frac{PB'}{PB}$$

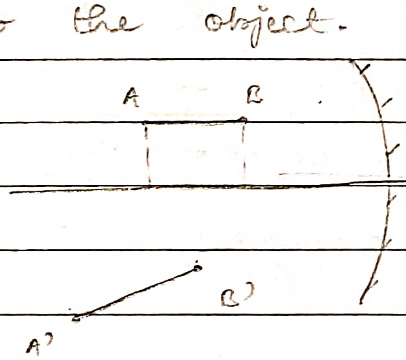
$$\Rightarrow \boxed{m = -\frac{v}{u}}$$

$$\Rightarrow \boxed{m = \left(\frac{f}{f-u}\right)}$$

For object inclined to optical axis, image is still a straight line because all pts. create image on reflected ray.



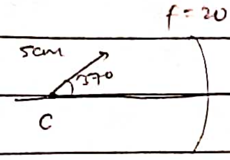
For object kept ∥ to optical axis, the image formed is a straight line not parallel to the object.



This straight line can be obtained by joining the images A' & B' of the end pts. A & B of the object m

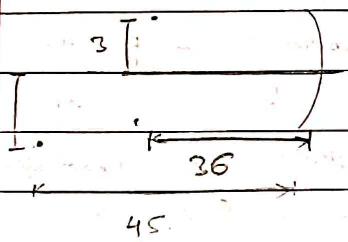
(+)	(−)
VO → RI	VO → VI
RO → VI	RO → RI
Mirror – Diff. side	Mirror – Same side
Lens – Same side	Lens – Diff. side
Erect image (Obj & Img. on same side of P.A)	Inverted image (Obj & Img. on diff side of P.A)

13/09/2023

Q.  Find length of image.

A. Pt at C will have image at C (bottom of the object)

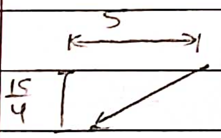
$$\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$$



$$\Rightarrow \frac{-1}{20} = \frac{1}{v} - \frac{1}{36}$$

$$\Rightarrow v = -45$$

$$m = -\frac{v}{u} \Rightarrow h_i' = (3)\left(\frac{45}{36}\right) = \left(\frac{15}{4}\right)$$



$$h_i = \sqrt{\left(\frac{15}{4}\right)^2 + 5^2} \quad \underline{25/4}$$

For a pt. object ; $\frac{1}{f} = \frac{1}{v} + \frac{1}{u}$

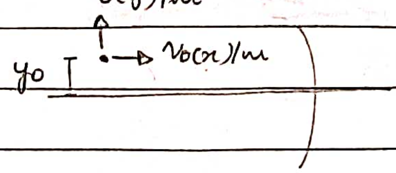
$$\xrightarrow{\frac{d}{dt}} \quad 0 = -\frac{1}{v^2}\frac{dv}{dt} - \frac{1}{u^2}\frac{du}{dt}$$

$$\Rightarrow \boxed{v_{i/m} = -\left(\frac{v^2}{u^2}\right) v_{o/m}}$$

$$\Rightarrow v_{i/m} = -m^2 v_{o/m}$$

(These vel. are along principal axis)

$v_{i(x)/m} = -m^2 v_{o(x)/m}$

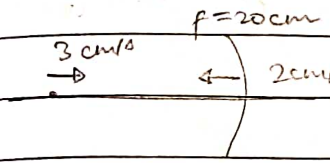


$\frac{y_i}{y_o} = m = \frac{f}{f-u}$

$\Rightarrow y_i = y_o\left(\frac{f}{f-u}\right) \xRightarrow{\frac{d}{dt}} \frac{dy_2}{dt} = \left(\frac{f}{f-u}\right)\frac{d(y_o)}{dt} + y_o \frac{d}{dt}\left(\frac{f}{f-u}\right)$

$= m\frac{dy_o}{dt} + y_o \frac{f^2}{(f-u)^2}\frac{1}{f}\frac{du}{dt}$

$\Rightarrow \boxed{v_{i(y)/m} = m v_{o(y)/m} + \frac{y_o m^2}{f} v_{o(x)/m}}$

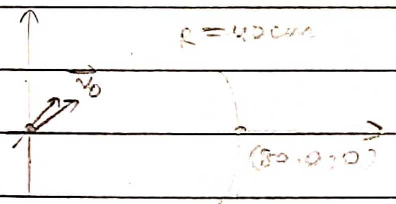
Q. 

Find v_i when $u = 30$cm

A. $v_{o/m} = 5$ cm/s $\Rightarrow v_{i/m} = -m^2 v_{o/m} = -\left(\frac{-20}{30-20}\right)^2 (5)$

$= -20$

$v_i = -20 + v_m = -20 - 2 = -22$ cm/s

Q. 

$\vec{v}_o = \langle 9 \quad 6 \quad 3 \rangle$

Find $\vec{v}_i$

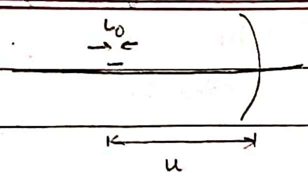
A. $v_{i(x)} = -m^2 v_{o(x)} = -\left(\frac{-20}{80-20}\right)^2 9 = -1$

$v_{i(y)} = m v_{o(y)} + (0) m^2 v_{o(x)} = \left(\frac{-1}{3}\right)(6) = -2$

$v_{i(z)} = m v_{o(z)} + (0) m^2 v_{o(z)} = \left(\frac{-1}{3}\right)(3) = -1$

$\vec{v}_i = \langle -1 \quad -2 \quad -1 \rangle$

Q. (Small object)



Find L_i

A. $L_o \rightarrow du$ $\qquad v_i = -m^2 v_o \Rightarrow dv = -m^2 du$

$L_i \rightarrow dv$ $\qquad \Rightarrow \underline{L_i = -m^2 L_o}$

Q. Pin placed $\perp$ to P.A. $\quad h_{i(1)} = h_{i(2)}$ at u_1 & u_2

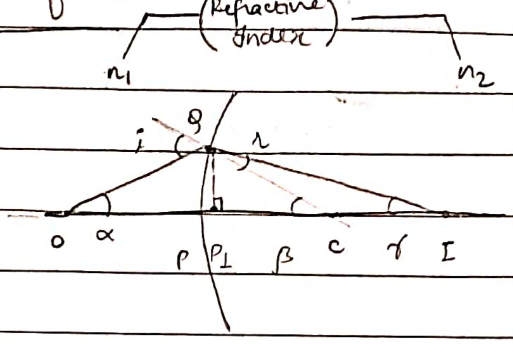
Find f

A. $h_{i(1)} = h_{i(2)} \Rightarrow m_1 = m_2 \Rightarrow \dfrac{f}{f-u_1} = \dfrac{f}{f-u_2} \Rightarrow \underline{u_1 = u_2}$

OR

$m_1 = -m_2 \Rightarrow \dfrac{f}{f-u_1} = \dfrac{f}{u_2-f} \Rightarrow f = \left(\dfrac{u_1+u_2}{2}\right)$

→ Refraction through Curved Surface



$i = (\alpha + \beta)$

$\beta = \gamma + r$

$\Rightarrow r = (\beta - \gamma)$

$n_1 \sin i = n_2 \sin r \sim n_1 i = n_2 r \Rightarrow n_1(\alpha+\beta) = n_2(\beta - \gamma)$

$\Rightarrow n_1 \alpha + n_2 \gamma = \beta(n_2 - n_1)$

$\Rightarrow n_1 \dfrac{QP_\perp}{OP_\perp} + n_2 \dfrac{QP_\perp}{IP_\perp} = \dfrac{QP_\perp}{CP_\perp}(n_2 - n_1)$

$OP \rightarrow (-u)$

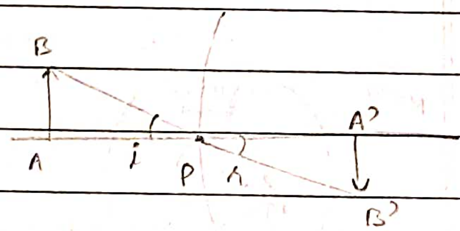
$IP \rightarrow v$

$CP \rightarrow R$

$\sim \dfrac{n_1}{OP} + \dfrac{n_2}{IP} = \dfrac{(n_2 - n_1)}{CP}$

$$\Rightarrow \boxed{\frac{n_2}{v} - \frac{n_1}{u} = \frac{(n_2 - n_1)}{R}}$$

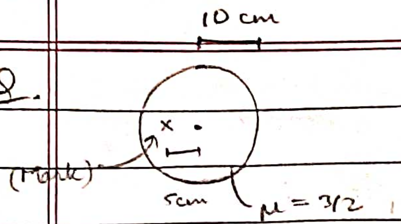
$m = -\dfrac{A'B'}{AB} = -\left(\dfrac{r}{i}\right)\left(\dfrac{\overbrace{A'P}^{v}}{\underbrace{AP}_{-u}}\right)$



$$\boxed{m = \left(\frac{n_1}{n_2}\right)\left(\frac{v}{u}\right)}$$

$i = \dfrac{AB}{PA}$ & $r = \dfrac{A'B'}{A'P}$

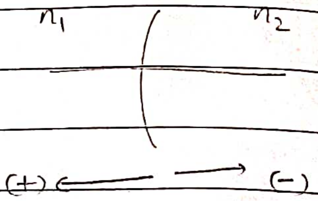
Q.



Find apparent depth of mark when viewed from
a) left side
b) right side

A. a) $\dfrac{1}{v_l} - \left(\dfrac{3/2}{-(10-5)}\right) = \dfrac{(-1/2)}{-10} \Rightarrow \dfrac{1}{v_l} = \dfrac{1}{20} - \dfrac{3}{10} = \left(\dfrac{-1}{4}\right)$

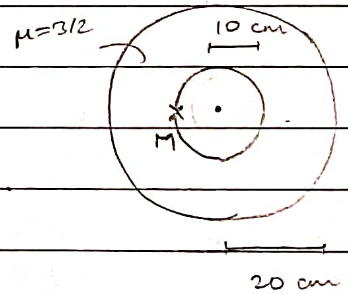
$\underline{v_l = -4}$



b) $\dfrac{1}{v_r} - \left(\dfrac{3/2}{-(10+5)}\right) = \dfrac{-(1/2)}{-10} = \dfrac{1}{v_r} = \dfrac{1}{20} - \dfrac{3}{30} = \dfrac{-1}{20}$

$\Rightarrow \underline{v_r = -20}$

Q.



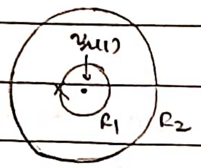
Find apparent depth of mark when viewed from
a) left side
b) right side

A. a) $\dfrac{1}{v_l} - \left(\dfrac{(3/2)}{-(20-10)}\right) = \dfrac{(-1/2)}{-20} \Rightarrow \dfrac{1}{v_l} = \dfrac{1}{40} - \dfrac{3}{20}$

$\underline{v_l = -8}$

b) R1: $\frac{(3/2)}{v_{r(1)}} - \left(\frac{1}{-20}\right) = \frac{(+1/2)}{-10} \Rightarrow \frac{1}{v_{r(1)}} = \frac{-1}{20} - \frac{1}{20} = \frac{-1}{10}$

$\Rightarrow v_{r(1)} = -10$



R2: $\frac{1}{v_{r(2)}} - \left(\frac{(3/2)}{-(10+(20-10))}\right) = \frac{(-1/2)}{(-20)} \Rightarrow \frac{1}{v_{r(2)}} = \frac{1}{40} - \frac{3}{40}$

$\Rightarrow \underline{v_{r(2)} = -20}$

★ Q.

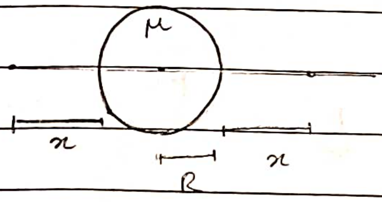
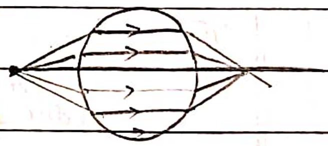


Image dist same when viewed from both sides. Find x.

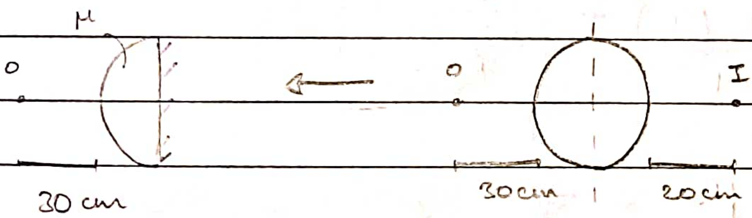
A. By reversibility of light, light must become $\parallel$ to P.A



$\Rightarrow v = \infty$

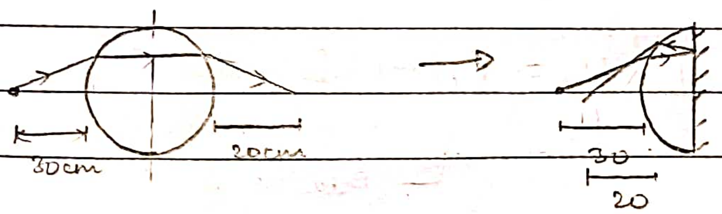
$\Rightarrow \frac{\mu}{\infty} - \frac{1}{(-x)} = \frac{(\mu-1)}{R} \Rightarrow \underline{x = \frac{R}{(\mu-1)}}$

★ Q.



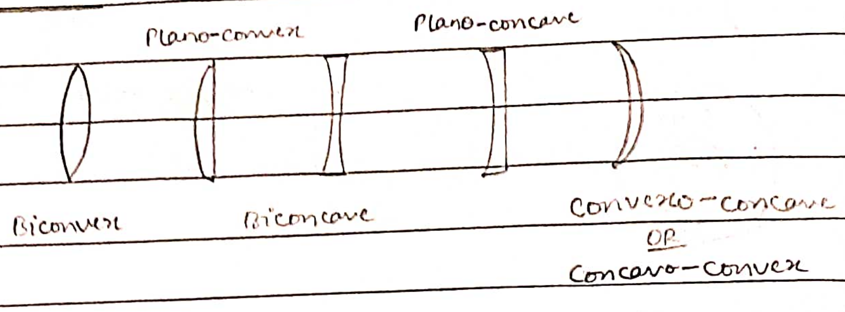
Find I for new config.

A.



∴ The image is still 20cm from the surface. Just the side is reversed

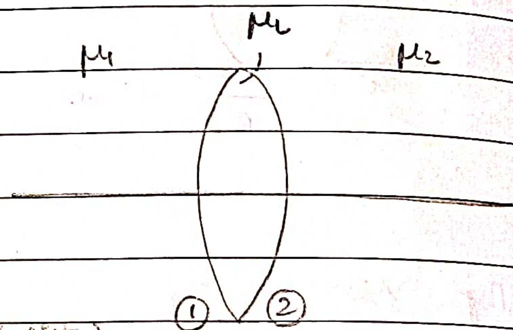
→ Thin lens



Concave lens ⟶ Diverging

Convex lens ⟶ Converging

(Normal behaviour $\mu_L > \mu_m$ (lens) (Medium))



$$\frac{\mu_L}{v_1} - \frac{\mu_1}{u} = \frac{\mu_L - \mu_1}{R_1}$$

Thin lens ⇒ $v_1 = u_2$

(img dist for ①) (obj dist for ②)

$$\frac{\mu_2}{v} - \frac{\mu_L}{u_2} = \frac{\mu_2 - \mu_L}{R_1}$$

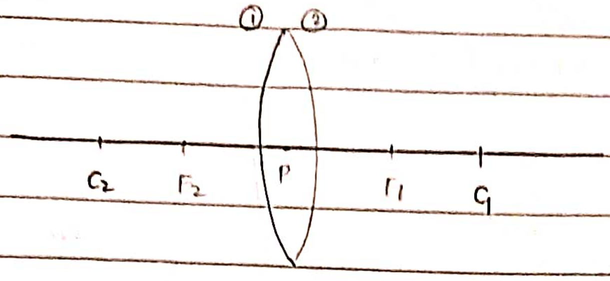
$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \left(\frac{\mu_L - \mu_1}{R_1}\right) - \left(\frac{\mu_L - \mu_2}{R_2}\right)}$$

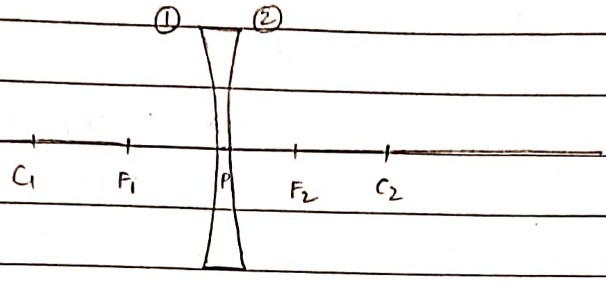
If $\mu_1 = \mu_2 = \mu_m$ ⇒ $\mu_m\left(\frac{1}{v} - \frac{1}{u}\right) = (\mu_L - \mu_m)\left[\frac{1}{R_1} - \right.$

$$\Rightarrow \boxed{\frac{1}{v} - \frac{1}{u} = (n-1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

(Relative μ)

$$n = \left(\frac{\mu_L}{\mu_m}\right)$$





f — F_2P (Dist. of F_2 from Pole / Optical Centre)

For $u = -\infty$, $v = f$ $\Rightarrow$ $\boxed{\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)}$

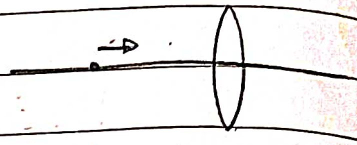
Lens Makers Formula

$\Rightarrow$ $\boxed{\frac{1}{f} = \frac{1}{v} - \frac{1}{u}}$

NOTE: If the medium on either sides of the lens is same, then it does not matter from which side the light enters. Image formed is unaffected. given pole remains fixed.

$$\boxed{m = \left(\frac{v}{u}\right)} = \left(\frac{f}{f+u}\right)$$

$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u}$$



$$\Rightarrow \quad 0 = -\frac{1}{v^2}\frac{dv}{dt} + \frac{1}{u^2}\frac{du}{dt}$$

$$\Rightarrow \quad \frac{dv}{dt} = \left(\frac{v^2}{u^2}\right)\frac{du}{dt} \quad \Rightarrow \quad v_{i/m} = m^2 v_{o/m}$$

(For velocities along Principal Axis)

In general, $\quad v_{i(x)/m} = m^2 v_{o(x)/m}$

$$\frac{y_i}{y_o} = \frac{v}{u} \quad \Rightarrow \quad y_i = y_o\left(\frac{v}{u}\right)$$

$$\frac{dy_i}{dt} = m\frac{dy_o}{dt} + y_o \frac{d}{dt}\left(\frac{f}{f+u}\right)$$

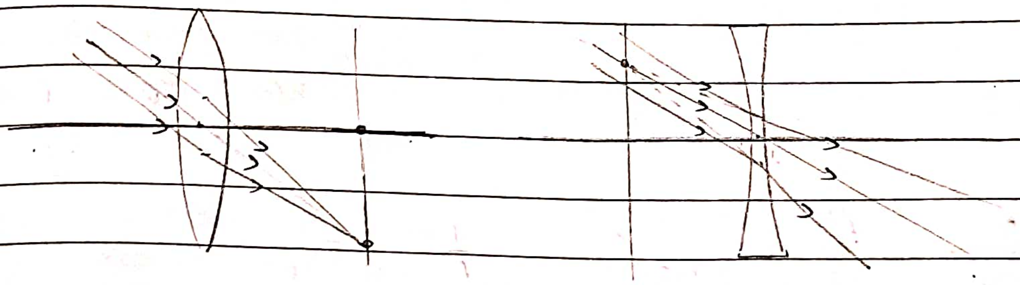
$$= m v_{o(y)/m} + y_o f\left(-\frac{1}{(f+u)^2}\frac{du}{dt}\right)$$

$$= m v_{o(y)/m} - \frac{y_o}{f}\underbrace{\left(\frac{f}{f+u}\right)}_{m}{}^{2} v_{o(x)/m}$$

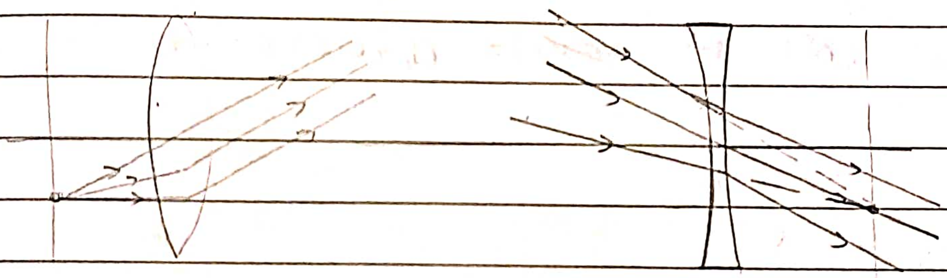
$$\boxed{v_{i(y)/m} = m v_{o(y)/m} - \frac{y_o}{f} m^2 v_{o(x)/m}}$$

(In mirror, this was (+) sign)

Focal Plane II — Image by || paraxial rays lies on FPII.



Focal Plane I — If object kept in FPI, rays || after refraction

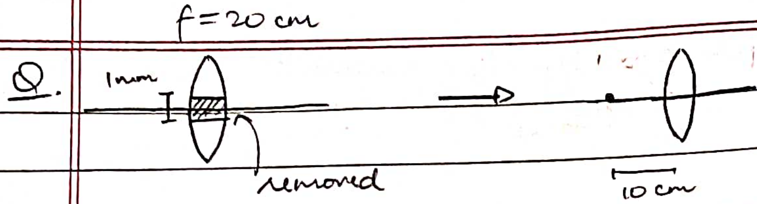


Cutting

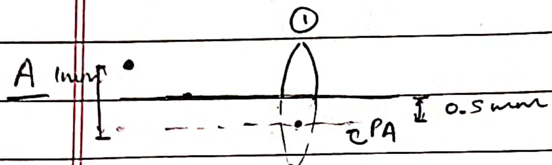
Focal length remains same

Principal axis remains same

Intensity / Brightness of image ↓

Q. 

How many images produced by the lens? Also find the dist b/w these images

A 

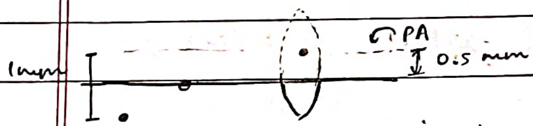
$$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{20} = \frac{1}{v} + \frac{1}{10}$$

$$\Rightarrow v = -20$$

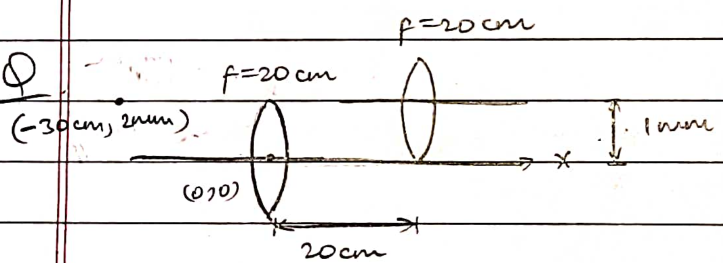
$$h_1 = \frac{v}{u}(0.5) = \underline{1 \text{ mm}}$$

$$h_2 = \frac{v}{u}(-0.5) = \underline{-1 \text{ mm}}$$

} 2 images



(Dist b/w images) = $(1-0.5)+(1-0.5) = \underline{1 \text{ mm}}$

Q 

Find coordinates of the final image produced by the given optical system

A $\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u_1}$ $\Rightarrow$ $v_1 = 60$ cm $h_i = (-2)(2) = -4$mm

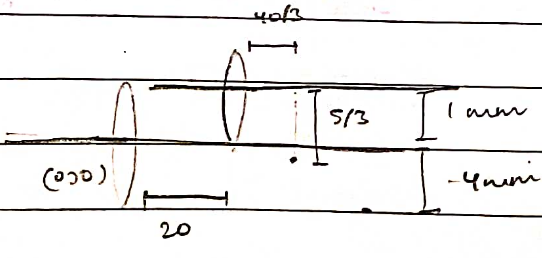
f_1 (20), u_1 (-30)

$\Downarrow$

$u_2 = 40$ cm

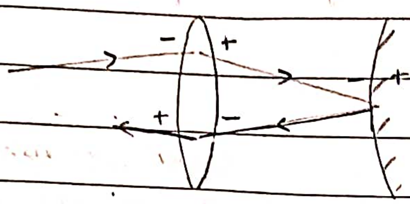
$$\frac{1}{f_2} = \frac{1}{v_2} - \frac{1}{u_2} \Rightarrow \frac{1}{20} = \frac{1}{v_2} - \frac{1}{40} \Rightarrow v_2 = 40/3 \text{ cm}$$

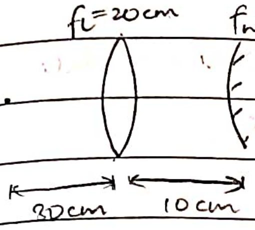
$$h_i' = \left(\frac{1}{3}\right)(4+1) = 5/3$$



$$\Rightarrow I \equiv \left(\frac{100}{3} \text{ cm}, \frac{-2}{3} \text{ mm}\right)$$

LENS- MIRROR COMBINATION





Q. Find posi. & nature of final image.

A. 1. $\frac{1}{f_L} = \frac{1}{v_1} - \frac{1}{u_1} \Rightarrow \frac{1}{20} = \frac{1}{v_1} - \left(\frac{-1}{30}\right) \Rightarrow v_1 = 60 \text{ cm}$

2. $u_2 = (60-10) = 80 \text{ cm}$ $\frac{1}{f_m} = \frac{1}{v_2} + \frac{1}{u_2} \Rightarrow \frac{1}{20} = \frac{1}{v_2} + \frac{1}{80}$

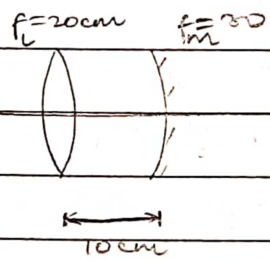
$\Rightarrow v_2 = \frac{100}{3}$

3. $u_3 = -10 - \frac{100}{3} = \frac{-130}{3}$

$\frac{1}{f_L} = \frac{1}{v_3} - \frac{1}{u_3} \Rightarrow \frac{1}{20} = \frac{1}{v_3} + \frac{3}{130} \Rightarrow v_3 = \left(\frac{260}{7}\right) \text{cm}$

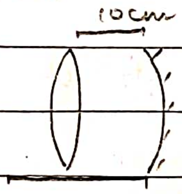
Lens: Obj. → Real img. (behind mirror)

Mirror (convex): Virtual obj. → Real img.

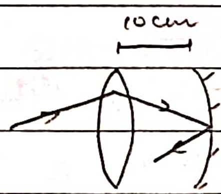


Q Find obj dist. for which final image coincides with the object.

A. CI



CII



$v_L = -50 \text{ cm} \Rightarrow -50 = \frac{u(20)}{u+20}$

$\Rightarrow u = \frac{-100}{7} \text{ cm}$

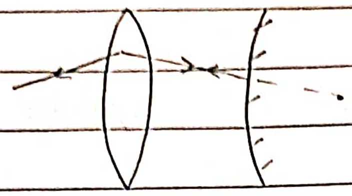
But convex lens can only create real image at $v > f$

⇓

Not possible as $\underset{(v)}{10cm} < \underset{(f)}{20cm}$

- <u>Image coinciding with Object</u>.

<u>CI</u>

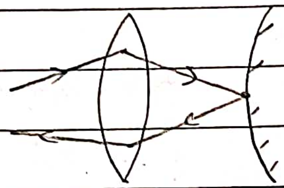


lens creates image at <u>C of mirror</u>

⇓

(Obj & Img. on same side of P.A)

<u>CII</u>



lens creates image at <u>Pole of mirror</u>

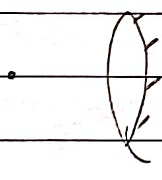
⇓

(Obj & Img. on diff side of P.A)

- <u>Silvering of Lens</u> — Silvered lens behaves as a combination of lens & mirror.

<u>Q</u>.

R = 40 cm



$\mu = 3/2$

At what dist should a pt. object be placed s.t its final image coincides with object.

<u>A</u>. It should appear to mirror as if rays coming from centre of curvature.

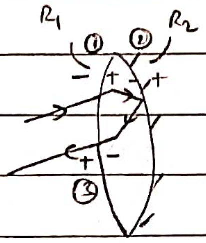
ie $v_1 = -40$

$$\frac{(3/2)}{-40} - \frac{1}{u} = \frac{(1/2)}{40}$$

$$\Rightarrow \frac{1}{u} = -\frac{3}{80} - \frac{1}{80} = -\frac{1}{20}$$

$\Rightarrow$ <u>$u = -20$</u> ← (effective Radius of curvature)

So, the optical system behaves as a concave mirror with $R = R_{eff}$.



In general,

$$\frac{\mu}{v_1} - \frac{1}{u} = \frac{(\mu-1)}{R_1}$$

$$\frac{2}{R_2} = \frac{1}{v_2} + \frac{1}{v_1}$$

$$\frac{1}{v} - \frac{\mu}{(-v_2)} = \frac{(1-\mu)}{(-R_1)}$$ ← (signs inverted due to change in dirxn)

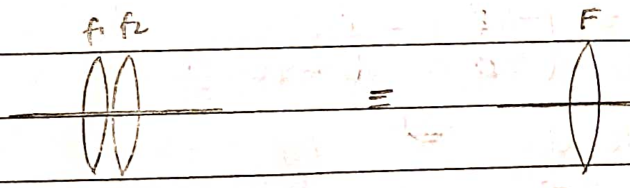
This is wrt to sign convention of ③

We will obtain the formula in sign convention of ① by putting $v \to (-v)$

$$\frac{1}{u} + \frac{1}{v} = \frac{2\mu}{R_2} - \frac{2(\mu-1)}{R_1} \longrightarrow \frac{1}{u} + \frac{1}{(-v)} = \frac{2}{R_2} - 2(\mu-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

$$\Rightarrow \boxed{\frac{1}{F} = \frac{-2}{f_L} + \frac{1}{f_m}}$$

• Combination of lenses –

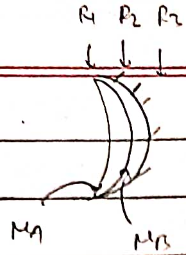


$$\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2}}$$

(eq. focal length)

Proof: $\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u}$, $\frac{1}{f_2} = \frac{1}{v} - \frac{1}{v_1}$ $\Rightarrow \left(\frac{1}{f_1} + \frac{1}{f_2}\right) = \left(\frac{1}{v} - \frac{1}{u}\right) = \frac{1}{F}$

Q.



$\mu_A = 1.6$, $\mu_B = 1.2$

$R_1 = 80$ cm, $R_2 = 40$ cm, $R_3 = 20$ cm

$u = -12$ cm. Find v

A. Method I

$$\frac{1}{f_1} = (0.6)\left[\frac{-1}{80} + \frac{1}{40}\right] = \frac{0.6}{80}$$

$$\frac{1}{f_2} = (0.2)\left[\frac{-1}{40} + \frac{1}{20}\right] = \frac{0.2}{40}$$

$$\frac{1}{f_{eq(L)}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{80}, \quad \frac{1}{F_{eq}} = -\frac{2}{f_{eq(L)}} + \frac{1}{f_m}$$

$$= -\frac{2}{80} + \left(\frac{-1}{10}\right)$$

$$= \frac{-1}{8} \Rightarrow F_{eq} = -8$$

$$\frac{1}{F_{eq}} = \frac{1}{v} + \frac{1}{u}$$

$$\Rightarrow \frac{-1}{8} = \frac{1}{v} + \left(\frac{-1}{12}\right)$$

$$\Rightarrow v = -24 \text{ cm}$$

Method II :

(If obj. kept at left, optical sys. should make img. at C of mirror)

$$\frac{1.6}{v_1} - \frac{1}{R} = \frac{1.6-1}{-80}$$

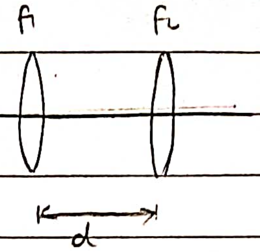
$$\frac{1.2}{-20} - \frac{1.6}{v_1} = \frac{1.2-1.6}{(-40)}$$

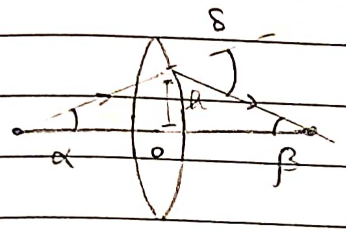
$$\frac{1}{R} - \frac{6}{800} = \left(\frac{-1.2}{20}\right) - \left(\frac{1.2-1.6}{-40}\right)$$

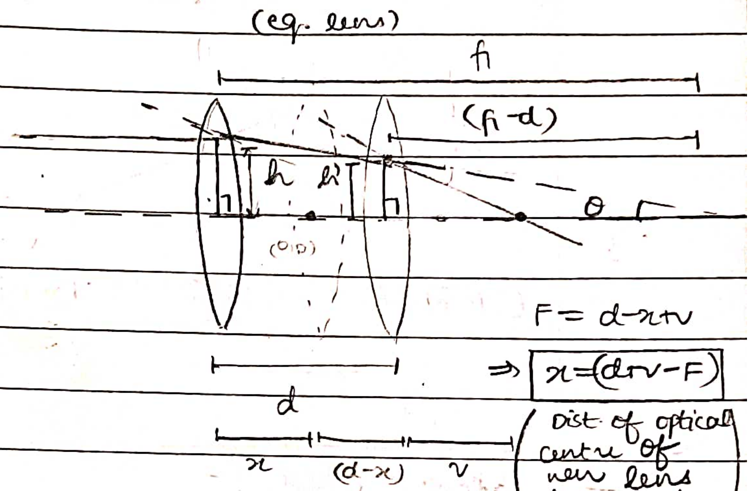
$$\Rightarrow R = -16 \text{ cm} \Rightarrow f_{eff} = -8 \text{ cm}$$

So, $\frac{-1}{8} = \frac{1}{v} + \left(\frac{-1}{12}\right) \Rightarrow v = -24$ cm

$$\boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$







$F = d - x + v$

$\Rightarrow \boxed{x = (d+v-F)}$ (Dist. of optical centre of new lens from L1)

$\delta = \alpha + \beta$

$= \frac{h}{-u} + \frac{h}{v}$

$= \frac{h}{f}$

$\delta = \delta_1 + \delta_2 \Rightarrow \frac{h}{F} = \frac{h}{f_1} + \frac{h'}{f_2}$

$\frac{1}{f_1} = \frac{1}{v_1} - \frac{1}{u}$ & $\frac{1}{f_2} = \frac{1}{v} - \frac{1}{(v_1-d)}$

$\Rightarrow \boxed{h\left(\frac{1}{F} - \frac{1}{f_1}\right) = \frac{h'}{f_2}}$

Taking $u = -\infty$, after passing both the lenses, ray focuses at eff. focal pt.

$\Rightarrow \frac{1}{f_1} = \frac{1}{v_1} \Rightarrow \frac{1}{f_2} = \frac{1}{v} - \frac{1}{(f_1-d)} \Rightarrow \frac{1}{v} = \frac{1}{f_2} + \frac{1}{(f_1-d)}$

Using similar Δs, $\frac{h'}{f_1-d} = \frac{h}{f_1} \Rightarrow \frac{h'}{h} = \left(\frac{f_1-d}{f_1}\right)$

$\frac{1}{F} - \frac{1}{f_1} = \frac{1}{f_2}\left(\frac{1-d}{f_1}\right) \Rightarrow \boxed{\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$

- Power of Lens - $(\mu_L - \mu_m)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$

$$\Rightarrow \boxed{P = \frac{\mu_m}{f}}$$

SI Unit : Dioptre

OPTICAL INSTRUMENTS

→ Human Eye

Focal length max when eye is relaxed

- Near Pt. - Nearest dist. of distinct vision
  $D = 25$ cm

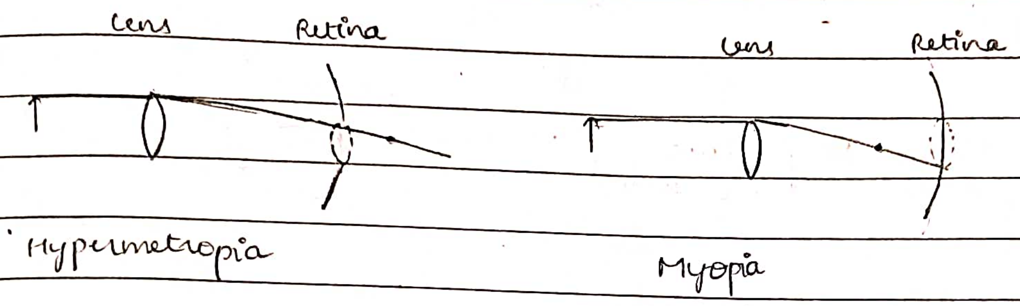
- Far Pt - Farthest dist of distinct vision

- Limit of Resolution - Min angle that must be subtended by an object on the eye in order to be visible.

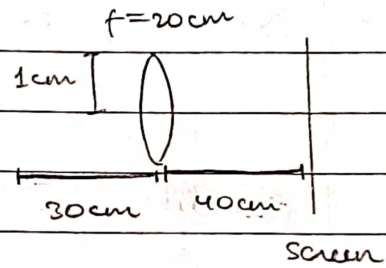
For eye $l_R = 1' = 1/60^\circ$

When N.P $\uparrow$ $\Rightarrow$ Long-sightedness (Hypermetropia)

F.P $\downarrow$ $\Rightarrow$ Near-sightedness (Myopia)

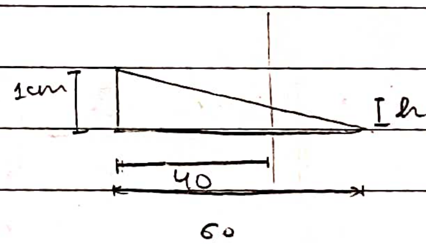


Q. Find the radius of bright region on the screen.



A. $v = \frac{uf}{u+f} = \frac{(-30)(20)}{-30+20} = \underline{60\,cm}$

$\frac{h}{60-40} = \frac{1}{60} \Rightarrow \underline{h = 1/3\ cm}$



Dist upto which the person can see

W/O glasses $\longrightarrow$ v

* With glasses $\longrightarrow$ u

* If dist upto which person wants to see (i.e with glasses) not given, to remove defect

N.P $\longrightarrow$ 25 cm

F.P $\longrightarrow$ ∞

Q. Long-sighted person N.P = 50 cm.
Find power on lens req. to remove defect.

A. $u = -25$, $v = -50$ $\Rightarrow$ $\frac{1}{f} = \frac{1}{v} - \frac{1}{u} = \frac{-1}{25} - \frac{1}{50}$

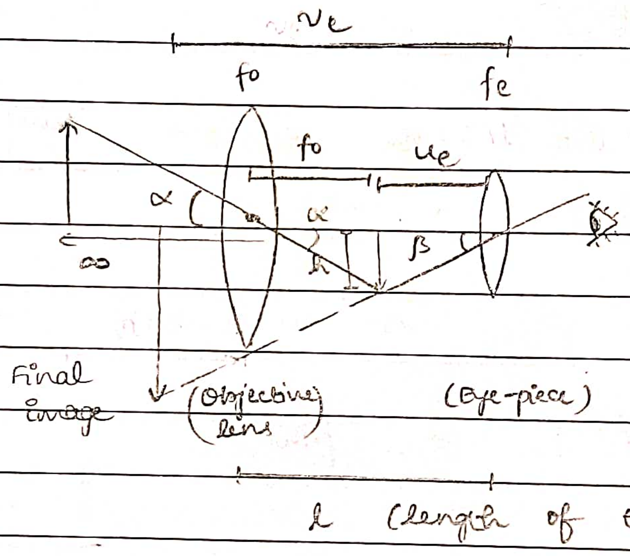
(If obj placed here) $\Rightarrow$ (Make img. here)

$= \frac{1}{50} = \underline{2D}$

→ <u>Astronomical Telescope</u>

Magnifying Power/ Magnification (for instrument) $= \left(\frac{\angle \text{ subtended on eye with i}}{\angle \text{ subtended on naked eye}} \right.$

* for calculating α, we assume obj. to be at
∞ for telescopes
D for microscope



since $l << \infty$ $\Rightarrow$ ∠ w/o instrument at eye $\sim \alpha$
∠ with instrument on eye $\sim \beta$

since $\alpha, \beta <<$ $\Rightarrow$ $\alpha = \frac{h}{f_o}$ & $\beta = \frac{h}{u_e}$

$\Rightarrow m = \frac{\beta}{\alpha} = \left(\frac{f_o}{u_e}\right)$

$$\frac{1}{f_e} = \frac{1}{(-v_e)} - \frac{1}{(-u_e)} \Rightarrow \frac{1}{u_e} = \frac{1}{v_e} + \frac{1}{f_e}$$

length of telescope

$$\Rightarrow \boxed{m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{v_e}\right)} , \quad \boxed{l = f_o + u_e}$$

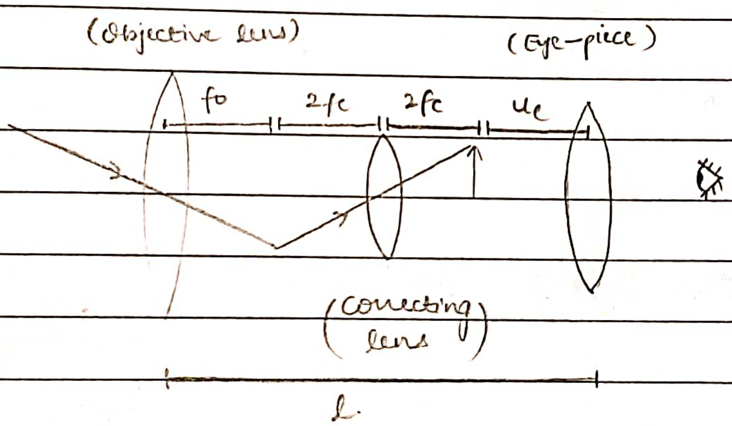
(v_e to be put here w/o sign)

For m_{min}, $v_e = \infty \Rightarrow m = \frac{f_o}{f_e}$
(normal vision/ relaxed eye)

m_{max}, $v_e = D \Rightarrow m = \frac{f_o}{f_e}\left(1 + \frac{f_e}{D}\right)$

- Terrestial Telescope

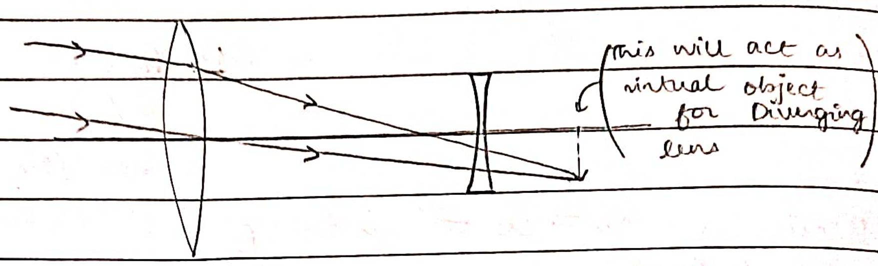
(Astronomical Telescope + correcting Lens)

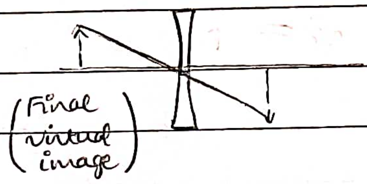


In astronomical telescope, image is seen upside down. To correct this, a correcting lens is inserted b/w Objective lens & Eyepiece

$$\boxed{l = (u_e + f_o) + 4f_e}$$

- Galilean Telescope

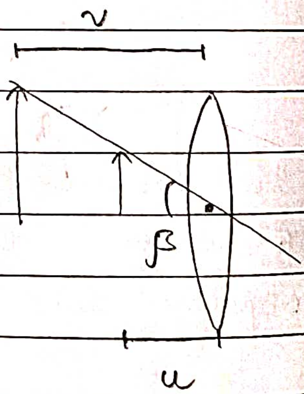




Formulae derived earlier still valid

→ Microscope

- Simple (Magnifying Glass)



$\alpha = \frac{h}{D}$ $\quad$ $m = \frac{\beta}{\alpha}$

$\beta = \frac{h}{u}$ $\quad \Rightarrow \boxed{m = \frac{D}{u}}$

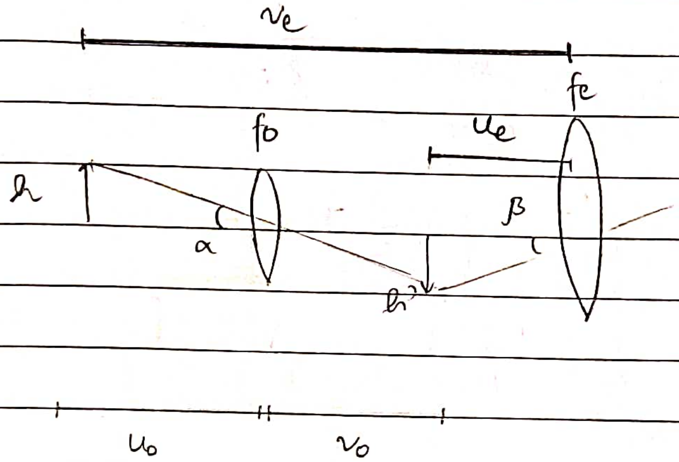
$\frac{1}{f} = \frac{1}{(-v)} - \frac{1}{(-u)} \Rightarrow \frac{1}{u} = \frac{1}{f} + \frac{1}{v}$

$\Rightarrow \boxed{m = \frac{D}{v} + \frac{D}{f}}$

For m_{min}, $v = \infty$ $\Rightarrow$ $m = D/f$
(normal vision/ relaxed eye)

m_{max}, $v = D$ $\Rightarrow$ $m = 1 + D/f$

- Compound



$$\alpha = \frac{h}{D}, \quad \beta = \frac{h'}{u_e} \Rightarrow m = \frac{\beta}{\alpha} = \left(\frac{h'}{h}\right)\left(\frac{D}{u_e}\right) \Rightarrow m = \frac{v_o D}{u_o u_e}$$

$$\frac{1}{f_e} = \frac{1}{(-v_e)} - \frac{1}{(-u_e)} \Rightarrow \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{v_e}$$

$$\Rightarrow \boxed{m = \frac{v_o}{u_o}\left(\frac{D}{v_e} + \frac{D}{f_e}\right)} = m_o \cdot m_e$$

For m_{min}, $v_e = \infty$ $\Rightarrow$ $m = \frac{v_o D}{u_o f_e}$
(normal vision/ relaxed eye)

m_{max}, $v_e = D$ $\Rightarrow$ $m = \frac{v_o}{u_o}\left(1 + \frac{D}{f_e}\right)$

If only f_o, f_e, l given, $v_o \sim l$

$$\frac{1}{f_o} = \frac{1}{v_o} - \frac{1}{(-u_o)} \sim \frac{1}{f_o} = \frac{1}{l} + \frac{1}{u_o}$$

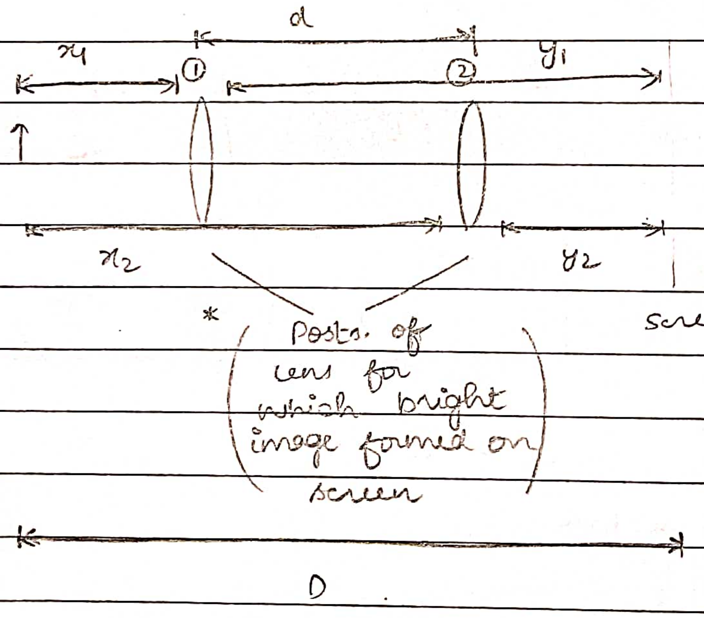
$$\Rightarrow \frac{l}{u_o} = \left(\frac{l}{f_o} - 1\right)$$

$$\Rightarrow m_{max} = \frac{v_o}{u_o}\left(1+\frac{D}{f_e}\right) \sim \frac{l}{u_o}\left(1+\frac{D}{f_e}\right)$$

$$= \left(\frac{l}{f_o} - 1\right)\left(1+\frac{D}{f_e}\right)$$

$$\sim \frac{l}{f_o}\left(1+\frac{D}{f_e}\right) \quad [l >> f_o]$$

→ Newton's Displacement Method
(for calculating f of convex lens)



* lens is continuously slid from ① to ②

By reversibility of light, $x_1 = y_2$ & $y_2 = x_1$

Since $x_1 + y_1 = D$ & $x_2 - x_1 = d$ $\Rightarrow$ $x_1 = \left(\frac{D-d}{2}\right)$

$\Rightarrow y_1 - x_1 = d$

$y_1 = \left(\frac{D+d}{2}\right)$

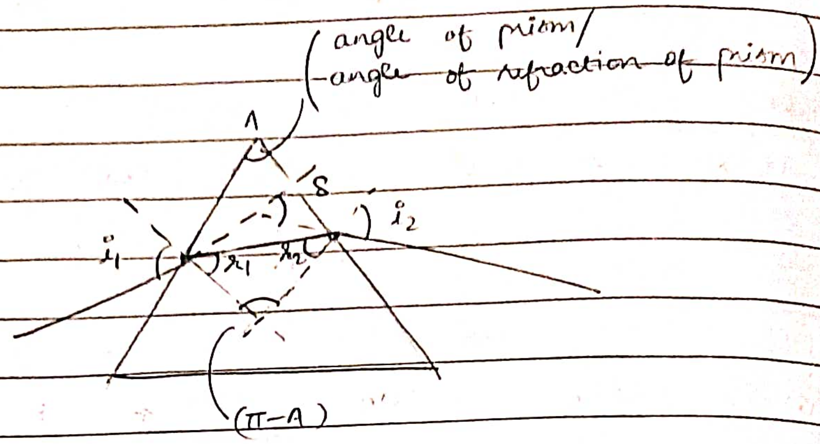
$\frac{1}{f} = \frac{1}{v} - \frac{1}{u} \Rightarrow \frac{1}{f} = \frac{2}{D+d} + \frac{2}{D-d}$

$\Rightarrow \boxed{f = \left(\frac{D^2 - d^2}{4D}\right)}$

$\because d^2 \geq 0 \Rightarrow \underline{f \leq D/4}$ (Condn for Newton's Method to work)

$m_1 = \frac{y}{x}$ & $m_2 = \frac{x}{y}$ $\Rightarrow$ $\underline{m_1 m_2 = 1}$

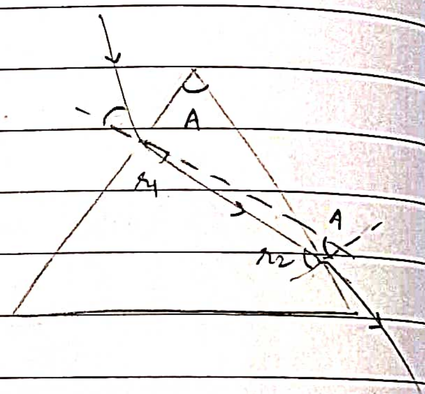
PRISM



$r_1 + r_2 + (\pi - A) = \pi \Rightarrow \underline{r_1 + r_2 = A}$

NOTE: This relⁿ only holds when light entering from base's side.

Otherwise,



$$\underline{A = (r_2 - r_1)}$$

We will derive the formulae only for the former case.

$$i_1 + i_2 + (\pi - A) + (\pi - \delta) = 2\pi$$

$$\Rightarrow \boxed{\delta = i_1 + i_2 - A}$$

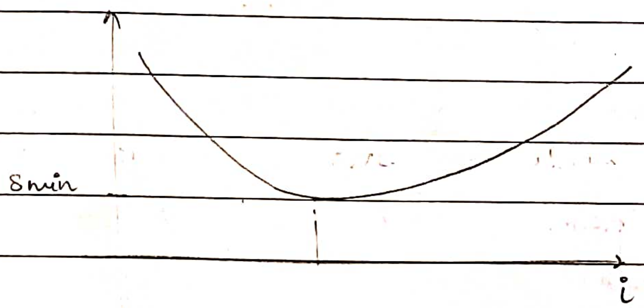
Observe, if light had entered with i_2 instead, it would have emerged out with i_1

$\Rightarrow \exists \; 2 \; i \; \forall \; \delta$

For $\delta = \delta_{min}$, $i_1 = i_2 = i \Rightarrow r_1 = r_2 = r$

$\Rightarrow r = \left(\frac{A}{2}\right)$

in isosceles prism, ray is $\parallel$ to base



* graph is not necessarily symmetrical

$\delta_{min} = i + i - A \Rightarrow i = \frac{\delta_{min} + A}{2}$

$\mu = \frac{\sin i}{\sin r} \Rightarrow \boxed{\mu = \frac{\sin\left(\frac{A}{2} + \frac{\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}}$

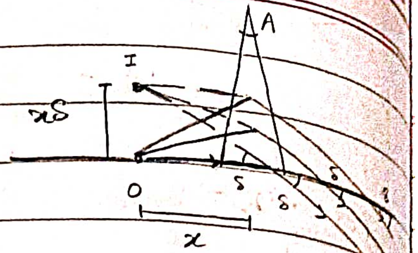
For small A, $\mu = \frac{A + \delta_{min}}{A} \Rightarrow \delta_{min} = (\mu - 1)A$

(Prism Approximation)

For paraxial rays, $\delta = \delta_{min} \Rightarrow \delta = (\mu - 1)A$

- Image formation by prism

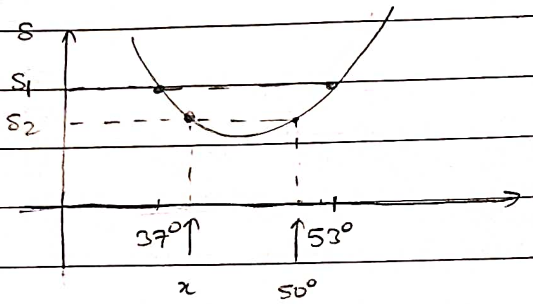
Shifting up by $x\delta$



★ Q. $i_1 = 53°$, $i_2 = 37°$. If $i_1 = 50°$, what can i

A) $35°$ B) $38°$ C) $40°$ D) $42°$

A.



Data implies

$\delta_{37°} = \delta_{53°} = (37°+53°) - A$

$\delta_1 = 90° - A$

$\Downarrow$

i for $\delta_{min} \in [37°, 53°]$

$\Downarrow$

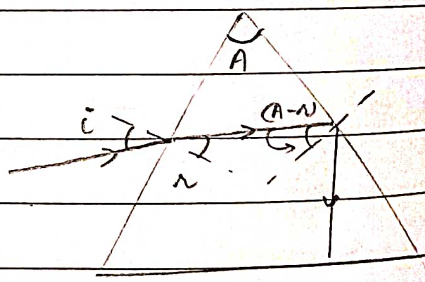
if $i_1 = 50° \Rightarrow \exists\ i_2 \geq 37°$ st

$\delta_{i_2} = \delta_{80°}$ $\Big\} \Rightarrow$ (B)

But since $\delta_1 > \delta_2$

$\Rightarrow 90° - A > i_2 + 50° - A \Rightarrow i_2 < 40°$

→ TIR from Prism



For TIR, $(A - r) \geq \theta_c$

For last ray suffering TIR,

$(A-r)_{max} \Rightarrow r_{min} \Rightarrow r = 0$ (Normal Incidence)

$\Rightarrow A - 0 \geq \theta_c$

$\Rightarrow \sin A \geq \sin\theta_c = \frac{1}{\mu}$

$\Rightarrow \boxed{\mu \geq \frac{1}{\sin A}}$

For first ray suffering TIR, $(A-r)_{min} \Rightarrow r_{max} \Rightarrow r = \theta_c$

(by reversibility of light)

(Grazing incidence)

$A - \theta_c \geq \theta_c \Rightarrow \theta_c \leq A/2$

$\Rightarrow \sin\theta_c \leq \sin(A/2)$

$\Rightarrow \boxed{\mu \geq \frac{1}{\sin(A/2)}}$

If the last ray suffers TIR (which it is least likely to out of all the rays), then all other rays will suffer TIR.

i.e No ray will emerge through 2nd face

22/09/2023

→ Dispersion

$\mu = \mu_0 + \frac{a}{\lambda} + \frac{b}{\lambda^2}$ $\lambda \to \left(\begin{array}{c}\text{wavelength}\\ \text{of light}\end{array}\right)$

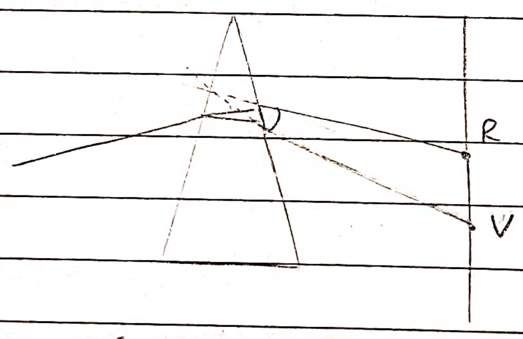
$\Rightarrow \lambda \downarrow \Rightarrow \mu \uparrow$

Average λ of light — 555 nm (Yellow-Green light)

$\mu_y = 1.5$

$\mu_v > \mu_y > \mu_R$

if only μ_v & μ_R given, $\mu_y = \left(\frac{\mu_v + \mu_R}{2}\right)$
can be considered



Angular dispersion $= \delta_v - \delta_R$

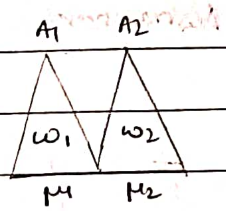
$= (\mu_v - \mu_R) A$

Dispersive Power $(\omega) = \frac{\text{Angular Dispersion}}{\text{Avg. Deviation}} = \frac{(\mu_v - \mu_R) A}{(\mu_y - 1) A}$

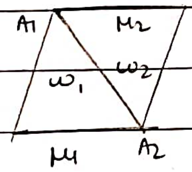
$$\boxed{\omega = \frac{\mu_v - \mu_R}{\mu_y - 1}}$$

So, ω is a property of glass

Alike prism $\Rightarrow$ $\delta = \delta_1 + \delta_2$



unlike prism $\Rightarrow$ $\delta = \delta_1 - \delta_2$



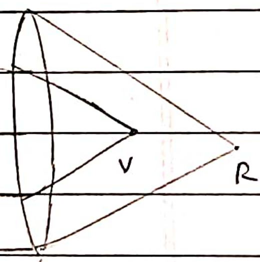
if $\delta = 0$ $\Rightarrow$ $(\mu_1 - 1)A_1 - (\mu_2 - 1)A_2 = 0$
(Dispersion w/o deviation)

if dispersion $= 0$ $\Rightarrow$ $\omega_1(\mu_1 - 1)A_1 - \omega_2(\mu_2 - 1)A_2 = 0$
(Deviation w/o dispersion)

- Chromatic Aberration –

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

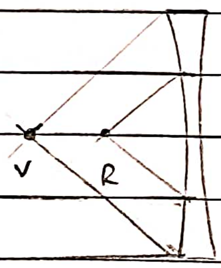
$$\Rightarrow f \propto \left(\frac{1}{n-1}\right)$$



$\lambda \downarrow \Rightarrow n \uparrow \Rightarrow f \downarrow$

$f_R - f_V > 0$

$$\omega = \frac{\Delta f}{f} = \frac{\left(\frac{c}{\mu_R - 1}\right) - \left(\frac{c}{\mu_V - 1}\right)}{\frac{c}{(\mu_y - 1)}}$$



$$= \boxed{\left(\frac{\mu_V - \mu_R}{\mu_y - 1}\right)}$$

$[(\mu_R - 1)(\mu_V - 1) \sim (\mu_y - 1)^2]$

$f_R - f_V < 0$

comb. of conv. & Div lens. s.t chromatic aberration

• Achromatic Doublet –

$$\frac{1}{F} = \frac{1}{f_1} + \frac{1}{f_2} \Rightarrow 0 = -\frac{df_1}{f_1^2} - \frac{df_2}{f_2^2} \Rightarrow \left(\frac{df_1}{f_1}\right)\left(\frac{1}{f_1}\right) + \left(\frac{df_2}{f_2}\right)\left(\frac{1}{f_2}\right)$$

$$\Rightarrow \boxed{\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0}$$

Condⁿ for A.D

Q. Acromatic doublet of 20 cm to be designed Glasses used ω_1 & ω_2 s.t $\frac{\omega_1}{\omega_2} = \frac{4}{5}$ Find f_1 & f_2

A. $\frac{1}{20} = \frac{1}{f_1} + \frac{1}{f_2}$ & $\frac{\omega_1}{f_1} + \frac{\omega_2}{f_2} = 0 \Rightarrow \frac{4}{f_1} + \frac{5}{f_2} = 0$

$\Rightarrow$ $f_2 = -5$ cm

$f_1 = 4$ cm